Energy Efficiency and Directed Technical Change: Implications for Climate Change Mitigation

Gregory Casey

December 2022

Abstract

I develop a directed technical change model of economic growth and energy efficiency in order to study the impact of climate change mitigation policies on energy use. I show that the standard Cobb-Douglas production function used in the environmental macroeconomics literature overstates the reduction in cumulative energy use that can be achieved with a given path of energy taxes. I also show that, in the model, the government combines energy taxes with research and development (R&D) policy that favors output-increasing technology – rather than energy efficiency technology – to maximize welfare subject to a constraint on cumulative energy use. In addition, I study energy use dynamics following sudden improvements in energy efficiency. Exogenous shocks that increase energy efficiency also decrease the incentive for subsequent energy efficiency R&D and actually increase long-run energy use relative to a world without the original shock. Subsidies for energy efficiency R&D, however, permanently alter R&D incentives and decrease long-run energy use.

Keywords Energy, Climate Change, Directed Technical Change, Growth

JEL Classification Codes H23, O33, O44, Q43, Q55

[*]Department of Economics, Williams College. *Email*: gpc2@williams.edu. I thank Lint Barrage, Oded Galor, and David Weil for their valuable guidance and insight throughout this project. For helpful feedback and suggestions, I am also grateful to Veronica Guerrieri (the editor), five anonymous referees, Daron Acemoglu, Ufuk Akcigit, Quamrul Ashraf, Joaquin Blaum, Stephie Fried, John Hassler, David Hemous, Garth Heutel, Peter Kruse-Andersen, Per Krusell, Derek Lemoine, Stelios Michalopoulos, Juan Moreno-Cruz, Gregory Nemet, Omer Ozak, David Popp, Soheil Shayegh, Sjak Smulders, Simone Valente, and Fabrizio Zilibotti, as well as seminar participants at SED 2021, the CESifo climate and energy economics workshop 2019, Econometric Society winter meeting 2019, SURED 2018, AERE 2017, NAREA 2017, AAEA 2017, SEA 2017, Amherst College, Brown University, Hamilton College, the Institute for International Economic Studies, Kobe University, Osaka University, SUNY Albany, University of Copenhagen, University of Virginia, University of Warwick, and Williams College. I thank Christophe McGlade for insights into the extraction cost data and Daniel Renwick for excellent research assistance. Any errors are my own.

1 Introduction

There is growing interest in using economic policy to reduce or eliminate carbon dioxide emissions. Aggregate energy efficiency – the ratio of gross domestic product to total energy use – is an important determinant of these emissions (e.g., Raupach et al., 2007; Peters et al., 2017). How will aggregate energy efficiency respond to policies designed to mitigate climate change? Many mitigation policies, like carbon taxes, will raise the tax-inclusive price of energy. Existing evidence suggests that increases in the price of energy have little immediate impact on energy efficiency, but they significantly increase energy efficiency over longer time horizons (Atkeson and Kehoe, 1999). Hassler et al. (2012, 2021a) show that directed technical change can explain these facts. In the quantitative climate-economy literature, many leading studies argue that adding energy to a Cobb-Douglas production function with exogenous technical change is an appropriate stand-in for directed technical change when modeling the reaction of energy efficiency to environmental policy interventions (e.g., Golosov et al., 2014; Hassler et al., 2016, 2021b; Barrage, 2020). This Cobb-Douglas approach, which has not been subjected to quantitative scrutiny, is consistent with the long-run data on energy prices and energy efficiency, but not the short-run data.

In this paper, I develop a tractable model of economic growth and energy efficiency. I use the model to theoretically and quantitatively study the impacts of environmental policy interventions. Following Hassler et al. (2021a), directed technical change governs the demand for energy. For a given set of technologies, energy and non-energy inputs must be combined in fixed proportions. Capital good producers, however, respond to increases in the relative price of energy by lowering the energy input ratio through directed research and development (R&D) activity, which implies that the economy is more flexible in the long run. The cost of supplying energy increases with cumulative extraction. The model is consistent with short- and long-run data on energy prices, energy use, and energy efficiency in the United States, as well as global data on energy extraction costs. I calibrate the model to these data for the quantitative analyses.

I use the calibrated model to show that the argument in favor of the Cobb-Douglas approach is misguided, because it fails to account for transition dynamics. Since the directed technical change model accurately captures the low short-run elasticity of substitution observed in the data, energy use reacts more slowly to the introduction of an energy tax, when compared to a model with Cobb-Douglas production. This causes the two models to yield different predictions for cumulative energy use following an identical policy intervention, even when they have similar predictions for long-run flow energy use. The difference in cumulative energy use is important, because climate change is a function of the stock of carbon in the atmosphere, rather than the flow of emissions. To quantify the difference between the two models, I find the path of energy taxes that maximizes welfare in the Cobb-Douglas model subject to a constraint on cumulative energy use that is consistent with the Paris Agreement. With the same path of taxes, cumulative energy use over the next century is 13 percent higher in the directed technical change model.

I also investigate the policies that maximize welfare subject to the cumulative energy use constraint in the directed technical change model. Similar to the existing literature on substitution between clean and dirty sources of energy, I find that separate policy instruments correct separate market failures (e.g., Acemoglu et al., 2012, 2016; Golosov et al., 2014; Hart, 2019). Energy taxes correct the two externalities from energy use: increases in future extraction costs and depletion of the remaining 'energy budget' that can be used under the constraint. R&D policy corrects externalities from intertemporal knowledge spillovers. The government always uses directed R&D policy along a transition path, but R&D policy need not favor energy efficiency. Indeed, in the quantitative analysis, R&D policy actually promotes *non*-energy technology, which increases energy use. Energy efficiency R&D is always above the laissez-faire level, but this is achieved through energy taxes, and R&D policy only addresses knowledge spillovers, which are more valuable for non-energy technology.

I also use the model to study rebound following exogenous improvements in energy efficiency and the introduction of subsidies for energy efficiency R&D. Rebound occurs when economic behavior undoes some of the partial equilibrium reduction in energy use following a sudden improvement in energy efficiency. Following an unexpected and exogenous improvement in economy-wide energy efficiency, there is no rebound in the very short run. But, this 'cost-less technology shock' reduces the incentive for subsequent energy efficiency R&D and actually increases long-run energy use, relative to world without the original shock. This is an extreme form of rebound known as 'backfire'. These findings complement existing applied microeconomic studies that estimate the degree rebound over short time horizons and find that backfire is unlikely (Borenstein et al., 2015; Gillingham et al., 2016). Since subsidies permanently affect R&D incentives, they reduce flow energy use in the long run. This difference with the exogenous shock supports recent arguments that the rebound literature should focus more on 'policy-induced' improvements in energy efficiency (Gillingham et al., 2016; Fullerton and Ta, 2020).

Related Literature. This paper contributes to the quantitative macroeconomic literature on climate change by constructing a growth model focusing on final-use energy.

Final-use energy refers to energy sources that are used directly in the production process, like electricity and gasoline. *Primary energy* refers to energy sources that are extracted from the environment and exist earlier in the production process, like coal and oil. Existing studies on directed technical change and climate change focus on clean versus dirty sources of primary energy and do not consider energy efficiency as a separate type of technology that can be improved through R&D (e.g., Acemoglu et al., 2012, 2016; Fried, 2018; Hart, 2019; Lemoine, 2020b). Meanwhile, the literature on endogenous, but not directed, energy efficiency improvements focuses on the efficiency of the energy transformation sector (e.g., Popp, 2004; Bosetti et al., 2006). While both of these margins are important, the data strongly suggest that the overlooked margin of final-use energy efficiency is an important long-run driver of carbon emissions. Studies with exogenous technology frequently assume that final-use energy is combined with capital and labor in a Cobb-Douglas production function (e.g., Nordhaus and Boyer, 2000; Golosov et al., 2014; Barrage, 2020; Hassler et al., 2021b), but this is at odds with well-known data patterns (Atkeson and Kehoe, 1999; Hassler et al., 2021a). Microeconomic evidence that changes in energy prices affect the direction of R&D is presented by Popp (2002) and Aghion et al. (2016), among others.

This paper is also related to the literature on directed technical change and energy use, which focuses on questions of long-run sustainability in the presence of exhaustible resources (e.g., Di Maria and Valente, 2008; André and Smulders, 2014; Hassler et al., 2021a).[1] I build on this literature in several ways. Most importantly, I analyze climate change mitigation policies. As discussed in Section 2.3, I also consider an alternative model of primary energy supply, which is better able to match long-run patterns in energy use. The most closely related paper is that of Hassler et al. (2012, 2021a). While they do not analyze the impacts of policy, their findings are frequently cited to support the use of the Cobb-Douglas production function in climate change economics (e.g., Golosov et al., 2014; Hassler et al., 2016, 2021b; Barrage, 2020).

In a review of the long existing literature on rebound, Gillingham et al. (2016) stress the need for an analysis that incorporates directed innovation. I perform such an analysis. This complements existing studies that investigate rebound in dynamic models with exogenous technical change (e.g., Saunders, 1992, 2000; Chang et al., 2018; Hart, 2018)[2] and recent advances in the literature that derive analytic expressions for rebound in dis-aggregated,

[1] See also Smulders and De Nooij (2003), who introduced directed technical change to the environmental macroeconomics literature in a study examining the impact of exogenous changes in the quantity of flow energy use.

[2] Related analyses have been conducted with large computable general equilibrium (CGE) models (e.g., Barker et al., 2009; Turner, 2009).

static models (e.g., Lemoine, 2020a; Fullerton and Ta, 2020; Blackburn and Moreno-Cruz, 2021).

Roadmap. Section 2 discusses the empirical motivation. The model is presented and analyzed in Section 3, and the calibration is presented in Section 4. Section 5 reports the results of the quantitative analyses, and Section 6 concludes.

2 Empirical Motivation

2.1 The Importance of Final-Use Energy

Final-use energy efficiency has played a crucial role in reducing the carbon intensity of output in the United States. Consider the following decomposition:

$$\frac{CO_2}{Y} = \frac{CO_2}{E_p} \cdot \frac{E_p}{E_f} \cdot \frac{E_f}{Y}, \tag{1}$$

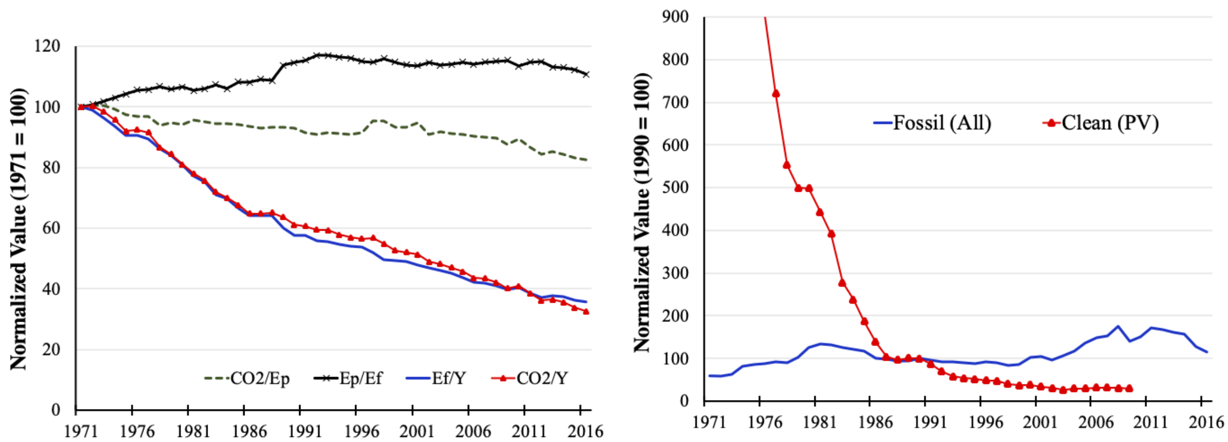
where CO_2 is annual carbon emissions, Y is gross domestic product, E_p is primary energy use (e.g., coal, oil), and E_f is final-use energy consumption (e.g., electricity, gasoline). The carbon intensity of primary energy, $\frac{CO_2}{E_p}$, captures substitution between clean and dirty sources of primary energy (e.g., coal versus solar). The efficiency of the energy sector, which transforms primary energy into final-use energy, is captured by $\frac{E_p}{E_f}$. For example, the ratio decreases when power plants become more efficient at transforming coal into electricity. Finally, $\frac{E_f}{Y}$ is the final-use energy intensity of output, which is the inverse of final-use energy efficiency. The ratio decreases, for example, when firms use less electricity to produce the same quantity of goods.

Panel (a) of Figure 1 plots each component of (1) from 1971-2016. Data are normalized to 1971 values.[3] The carbon intensity of output fell by almost 70 percent during this time period, and this decline is closely matched by the decline in the final-use energy intensity of output, $\frac{E_f}{Y}$. The carbon intensity of primary energy, $\frac{CO_2}{E_p}$, declined approximately 18 percent over this period. While this is a significant improvement for environmental outcomes, it is small compared to the overall change in the carbon intensity of output. Finally, the efficiency of the energy transformation sector, as measured by the inverse of $\frac{E_p}{E_f}$, actually declined roughly 11 percent over this period.[4,5]

[3] Appendix Section A describes the data and provides links to the original sources.

[4] This result is driven by differences in the efficiency of transformation across sources of primary energy, rather than technological regress.

[5] The results presented here are specific to the United States and may not generalize to other countries.



(a) Carbon Accounting

(b) Energy Prices

Figure 1: Panel (a) decomposes the decline in the carbon intensity of output in the United States using the identity $\frac{CO_2}{Y} = \frac{CO_2}{E_p} \cdot \frac{E_p}{E_f} \cdot \frac{E_f}{Y}$, where CO_2 is yearly carbon emissions, Y is GDP, E_p is primary energy, and E_f is final-use energy. Panel (b) plots the real price of fossil fuel energy and the real cost of generating electricity from solar energy over this period. Data are taken from the International Energy Agency (IEA), the Bureau of Economic Analysis (BEA), and Nemet (2006) via Nagy et al. (2013).

The carbon accounting evidence strongly suggests that final-use energy efficiency cannot be ignored when thinking about the determinants of carbon emissions. Without price data, however, it is difficult to know exactly how well existing trends capture the impact of environmental policy interventions that raise the price of carbon. A tax on carbon has two main effects. First, it will raise the price of carbon-intensive sources of primary energy relative to carbon-free sources of primary energy. Second, it will raise the price of final-use energy relative to non-energy factors of production. Panel (b) of Figure 1 demonstrates that the average real price of final-use energy derived from fossil fuels increased over this period. Unfortunately, a broad measure of the price of carbon-free energy is not available over this period or level of aggregation. Existing evidence, however, strongly suggests that the real price of carbon-free energy has been declining (e.g., Covert et al., 2016; Gillingham and Stock, 2018). To highlight this point, panel (b) also plots an estimate of the real cost of

The United States will be the focus of the quantitative analysis. To the best of my knowledge, earlier versions of this paper were the first to perform a carbon accounting exercise using (1). Previous studies focused on the Kaya Identity. These previous studies show that aggregate energy efficiency is the main driver of long-run trends in carbon emissions across a wide range of countries, but they do not consider the distinction between primary and final-use energy (e.g., Raupach et al., 2007; Peters et al., 2017). Independent work by Le Quéré et al. (2019) uses a related methodology to perform a carbon accounting analysis for 18 developed countries over the period 2005-2015. They find that, on average, reduced final-use energy consumption – a decrease in E_f – is the second biggest contributor to emissions reductions, with substitution between primary sources being the largest factor. They do not quantify the importance of final-use energy efficiency, E_f/Y. Reduced final-use energy consumption energy was responsible for five percent of U.S. CO_2 reductions over this period.

generating electricity from solar energy over this period.[6] Improvements in final-use energy efficiency were the dominant source of reductions in the carbon intensity of output, even during a time period when the price of carbon-free energy decreased relative to the price of fossil fuels. This result suggests that the margin of final-use energy efficiency will be important in understanding the impact of carbon taxes and other policies that raise the relative price of fossil fuels.

Motivated by these findings, this paper focuses on final-use energy efficiency and its role in climate change mitigation. It is important to note that none of the evidence presented in this section suggests that substitution between clean and dirty energy sources is unimportant. The quantitative analyses in this paper focus on energy use reductions that are necessary to meet environmental policy goals even in the presence of large-scale substitution toward carbon-free sources of primary energy.

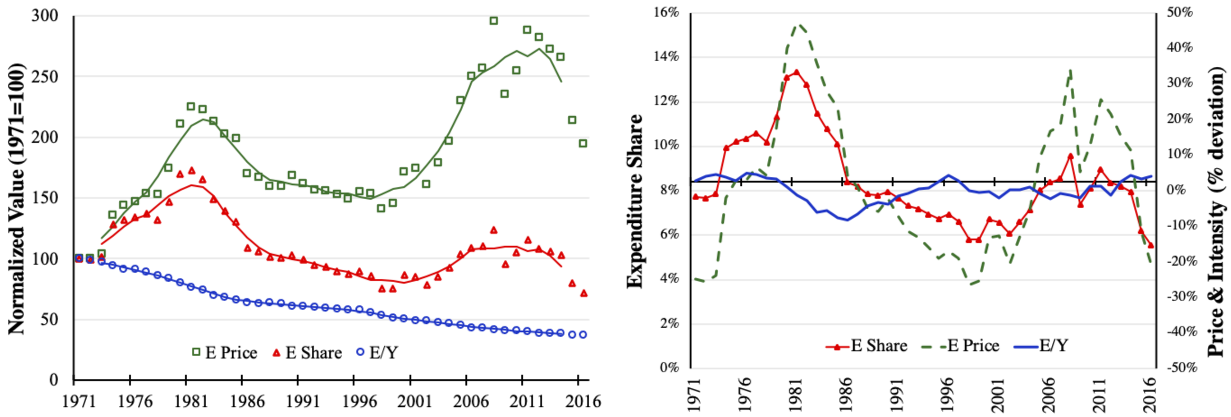
2.2 Energy Demand

Figure 2 summarizes evidence on the demand for energy. It shows the expenditure share of energy (E_{share}), the energy intensity of output (E/Y), and the average real price of final-use energy (p_E) in the United States from 1971-2016. The energy share of expenditure is total expenditure on final-use energy divided by GDP. These objects are related through the following identity: $E_{share} = p_E \cdot \frac{E}{Y}$. Panel (a) plots annual values, along with five-year moving averages, to highlight medium- and long-run trends. Panel (b) plots detrended annual values for E/Y and p_E against E_{share} to highlight short-run fluctuations.

The data indicate that the expenditure share, but not the energy intensity of output, reacts to short-run price fluctuations, suggesting that it is difficult to substitute between energy and non-energy inputs in the short run. Hassler et al. (2021a) provide a maximum likelihood estimate of the short-run elasticity of substitution between energy and non-energy inputs and find a value close to zero. As seen in both figures, the expenditure share can deviate from its long-run average by a substantial amount and for a significant period of time. Despite increasing prices, however, there is no long-run trend in the energy expenditure share. Hassler et al. (2021a) show that a directed technical change (DTC) model can recreate these facts (see also, Hart, 2013; André and Smulders, 2014).[7] I build on their work by using a

[6] The data are originally from Nemet (2006) and were accessed via the Performance Curve Database from the Sante Fe Institute (Nagy et al., 2013). These data are for illustrative purposes and will not be used in the quantitative analyses.

[7] Not all improvements in aggregate energy efficiency need to be driven by technical change. In particular, sectoral reallocation could potentially explain changes in aggregate energy use. Decomposition analyses



(a) Normalized Values

(b) Detrended

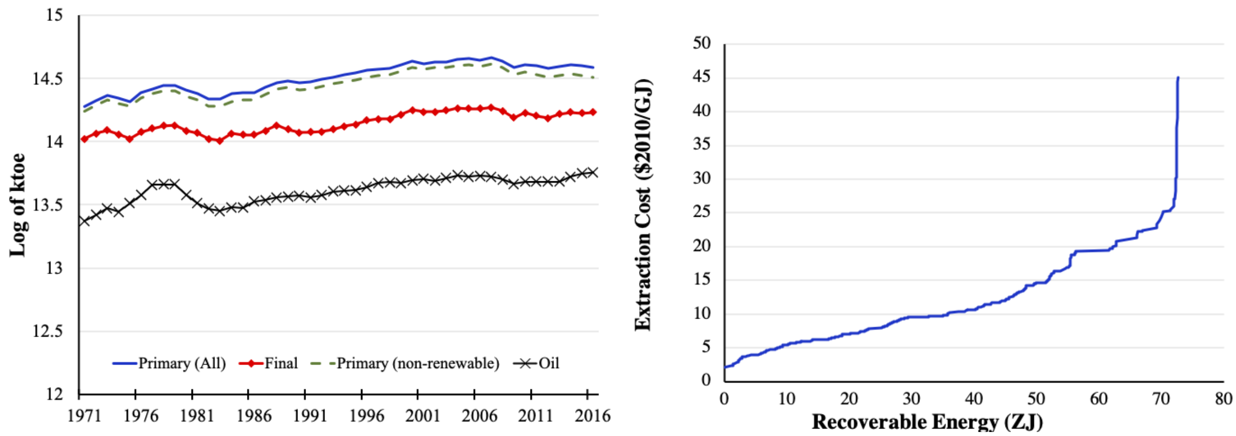
Figure 2: This figure shows the energy expenditure share (E_{share}), final-use energy intensity of output (E/Y), and average real energy price (p_E) in the United States from 1971-2016. These objects are related through the identity $E_{share} = p_E \cdot \frac{E}{Y}$. Panel (a) presents data normalized to 1971 values (markers) and 5-year moving averages (lines). Panel (b) presents detrended values for E/Y and p_E, along with the expenditure share. Trends are estimated via OLS, assuming a constant growth rate over the period. Data are taken from the Energy Information Administration (EIA) and the BEA.

DTC model to study the impacts of environmental policy.

2.3 Energy Supply

To differentiate between possible causes of rising energy prices, I now turn to trends in energy use. Studies with aggregate energy use almost always use one of two underlying models of energy supply to explain long-run trends in prices: optimal depletion of finite resources (e.g., Hotelling, 1931) or increasing extraction costs (e.g., Pindyck, 1978; Slade, 1982). Existing work on DTC and the environment focuses on the former (e.g., André and Smulders, 2014; Hassler et al., 2021a). Of the two approaches, however, only the increasing extraction cost model is consistent with aggregate evidence from the United States. If rising prices are driven by forward looking behavior and finite supplies, then energy use must decrease on the balanced growth path, which is when the energy expenditure share is constant. Figure

suggest that improvements in intra-sectoral efficiency, rather than reallocation, have been the key driver of falling energy intensity over this period (Sue Wing, 2008; Metcalf, 2008). Existing work suggests that there was a significant regime shift in both energy prices and energy efficiency improvements coinciding with the energy crisis of the early 1970s (e.g., Baumeister and Kilian, 2016; Fried, 2018; Hassler et al., 2021a). In the two decades prior to the crisis, energy prices were constant or decreasing, and decomposition analyses suggest that sectoral reallocation was the primary driver of falling energy intensity (Sue Wing, 2008). See Hart (2018) for a model focusing on earlier periods when trends in aggregate energy efficiency were driven by sectoral reallocation and energy prices were constant or decreasing.



(a) Energy Use

(b) Extraction Costs

Figure 3: Panel (a) shows aggregate energy use in the United States over the period 1971-2016. *ktoe* is kilotons of oil equivalent, a measure of energy content. Panel (b) presents estimates of the availability and extraction cost of fossil fuel resources. Original estimates are from McGlade and Ekins (2015b). Quantities are measured as zettajoules (ZJ) of final-use energy that can be extracted from available primary energy resources. Transformations from primary to final-use energy availability were performed using information from Rogner et al. (2012), the EIA, and the IEA.

3 shows that energy use has been increasing over the period of study. Thus, the aggregate data are inconsistent with a model where increasing prices are driven by scarcity rents.

Panel (b) of Figure 3 provides direct evidence for the existence of increasing extraction costs. It shows estimates of fossil fuel extraction costs and availability as of 2010 from McGlade and Ekins (2015b). The figure aggregates across the three sources of fossil fuels – oil, natural gas, and coal – and converts primary energy availability into final-use energy availability using average transformation rates.[8] Based on the evidence presented in Figure 3, I consider the case of increasing extraction costs, which allows for increasing energy use on the balanced growth path.

3 Model

3.1 Structure

3.1.1 Final Good Production

The model extends the standard endogenous growth production function to include energy use. Final good production is perfectly competitive, and the price of the final good is

[8]See Appendix Section A for more information on data sources, and Appendix Section C.1 for details on the construction of the figure.

normalized to one. To match the low short-run elasticity of substitution between energy and non-energy inputs, I consider a Leontief structure

$$Q_t = \int_0^1 \min\left[X_t(i)^\alpha (A_{N,t}(i)L_t)^{1-\alpha}, A_{E,t}(i)E_t(i)\right] \, di, \tag{2}$$

$$s.t. \quad A_{E,t}(i)E_t(i) \leq X_t(i)^\alpha \left(A_{N,t}(i)L_t\right)^{1-\alpha} \;\; \forall i, \tag{3}$$

where $\alpha \in (0,1)$. There is a continuum of capital goods, which are indexed by i. Also, Q_t is gross output at time t, $A_{N,t}(i)$ is the quality of capital good i, $X_t(i)$ is the quantity of capital good i, L_t is the aggregate (and inelastic) labor supply, $A_{E,t}(i)$ is the energy efficiency of capital good i, and $E_t(i)$ is the amount of energy devoted to operating capital good i. As in the standard endogenous growth production function, output is generated by a Cobb-Douglas combination of aggregate labor and a series of production processes, each of which uses a different capital good. Unlike the endogenous growth literature, each production process also requires energy to run. Thus, the usual capital-labor composite measures the maximum output that can be created using each production process, and the actual level of output depends on the amount of energy devoted to each process. I use $E_t \equiv \int_0^1 E_t(i)di$ to denote total energy use at time t. The notion of maximum output is captured by constraint (3).[9] The quality of the capital good, $A_{N,t}(i)$, improves its ability to produce output. The energy efficiency of the capital good, $A_{E,t}(i)$, lowers the amount of energy needed to operate at the maximum level.[10] I also deviate from the standard endogenous growth set by assuming that both types of technology are embodied in the capital goods, which helps keep the model tractable.

3.1.2 Energy Sector

Energy extraction costs are increasing in cumulative extraction. Energy is extracted from the environment using the final good. The increasing extraction costs incorporate two forces that govern long-run energy availability. First, they capture the increase in cost needed to extract conventional energy resources from harder-to-access areas. Second, they capture the increase in cost that may occur when a particular energy source is exhausted, necessitating

[9]In equilibrium, the final good producer will always choose inputs to set the arguments of the minimum function equal to each other, even in the absence of the constraint.

[10]Consistent with both the econometric and DTC literatures, improvements in non-energy technology, $A_{N,t}(i)$, as well as increases in capital or labor, raise energy requirements (e.g., Van der Werf, 2008; Hassler et al., 2021a).

a switch to a type of energy that is more difficult to extract.

Production is open access. This assumption is necessary to match data on long-run energy use, and it implies that energy suppliers do not internalize the impact of current extraction on future extraction costs. When examining the implications of the model, I also assume that the underlying energy supply limits are never reached. The infinite supply of energy and increasing extraction costs capture the existence of 'unconventional' energy sources, which have high extraction costs but are available in vast quantities.[11] The treatment of energy sources as infinite in potential supply also incorporates the abundance of coal, which is predicted to be the major driver of climate change (e.g., van der Ploeg and Withagen, 2012; Golosov et al., 2014).[12] Together, the vast quantities of coal and 'unconventional' energy sources imply that using too much fossil energy, rather than exhausting supply, is the relevant concern for policy (Covert et al., 2016).

The marginal cost of extraction, which will also be equal to the price, is given by

$$p_{E,t} \quad = \quad A_{V,t}^{-1}\bar{E}_{t-1}^{\psi}, \tag{4}$$

where $\bar{E}_{t-1}$ is cumulative energy ever extracted at the end of period $t-1$, and $A_{V,t}^{-1}$ captures the difference in the state of technology between the energy extraction and final good sectors. The parameter $\psi > 0$ gives the elasticity of energy extraction costs with respect to cumulative extraction. For simplicity, I assume that differential technological progress is given by the exogenous process

$$A_{V,t}^{-1} = (1 - g_{A_V})A_{V,t-1}^{-1}, \tag{5}$$

where $g_{A_V} < 1$, and focus on directed technical change in energy demand. If $g_{A_V} > 0$, then

[11]For example, Rogner et al. (2012) estimate a resource base of 4,900 – 13,700 exajoules (EJ) for conventional oil, compared with annual production of 416 EJ across all energy sources. Thus, constraints on availability of conventional oil sources may be binding. The ability to exhaust fossil fuel energy sources, however, appears much less likely when considering other options. The resource base for unconventional sources of oil is estimated to be an additional 3,750 – 20,400 EJ. Meanwhile, the resource bases for coal and natural gas (conventional and unconventional) are 17,300–435,000 EJ and 25,100 – 130,800 EJ, respectively. These estimates rely on projections regarding which resources will be profitable to extract from the environment. When considering the full range of energy sources that could become profitable to extract as resource prices tend towards infinity, the numbers grow even larger. In particular, such 'additional occurrences' are estimated to be larger than 1 million EJ for natural gas.

[12]Golosov et al. (2014) specify a finite amount of coal, but assume it is not fully depleted. Thus, it has no scarcity rent, although it does have an extraction cost. Oil, by contrast, is assumed to have no extraction cost, but does have a positive scarcity rent. Hart and Spiro (2011) survey the empirical literature and find little evidence that scarcity rents are a significant component of energy costs. They suggest that policy exercises focusing on scarcity rents will give misleading results.

technological progress is faster in the extraction sector.

The law of motion for the stock of extracted energy is given by

$$\bar{E}_t = E_t + \bar{E}_{t-1}. \tag{6}$$

The fact that extraction costs are constant within each period is a useful simplification. As motivation, it is intuitive to consider the case where energy producers exploit new sources of energy in each period and the difficulty of extraction is constant within each source.[13]

3.1.3 Final Output

Final output is equal to gross output minus total energy extraction costs. Constraint (3) will always holds with equality, because the final good producer would never hire labor, rent capital, or purchase energy that went unused. Thus, final output is given by

$$Y_t \quad = \quad L_t^{1-\alpha} \int_0^1 \left[1 - \frac{p_{E,t}}{A_{E,t}(i)}\right] A_{N,t}(i)^{1-\alpha} X_t(i)^{\alpha} \; di. \tag{7}$$

This formulation further highlights the continuity between the production function used here and the standard approach in endogenous growth models. Output has the classic Cobb-Douglas form with aggregate labor interacting with a continuum of capital goods. The model developed in this paper extends the standard endogenous growth model by considering a broader notion of productivity, $\left[1 - \frac{p_{E,t}}{A_{E,t}(i)}\right] \cdot A_{N,t}(i)^{1-\alpha}$. Productivity is determined by two different types of embodied technology, as well as energy extraction costs. The functional form is driven by the fact that underlying production function is Leontief. In the long run, the updated technology index grows at a constant rate, and the model can explain all of the usual balanced growth facts.

3.1.4 Capital Goods and Research

Each type of capital good is produced by a single profit-maximizing monopolist in each period. This monopolist also undertakes in-house R&D to improve the embodied technological

[13] I use the model to investigate the effect of policies pursued in the United States. In this case, endogenous energy prices can be motivated in two ways. First, it is possible to think of the United States as a closed economy. Alternatively, one can imagine the policies being applied worldwide with the United States using a constant fraction of total energy. In robustness analyses, I also consider the case of exogenous energy prices, which implicitly treats the United States as a small open economy taking unilateral policy actions.

characteristics, $A_{N,t}(i)$ and $A_{E,t}(i)$. The R&D production function is given by

$$A_{J,t}(i) \;=\; A_{J,t-1}(i) + \eta_J R_{J,t}(i)^{1-\lambda} A_{J,t-1}, \quad J = N, E, \tag{8}$$

where $R_{J,t}(i)$ gives the quantity of R&D inputs assigned to characteristic J by firm i in period t, and $A_{J,t-1} \equiv \int_0^1 A_{J,t-1}(i)di$. I also define $R_{J,t} \equiv \int_0^1 R_{J,t}(i)di$. There are decreasing returns to R&D within a period, governed by $\lambda \in (0,1)$, and $\eta_J > 0$ gives the exogenous component of research efficiency. Monopolists make decisions to maximize single period profits, implying that they ignore the intertemporal spillover in R&D productivity. All firms start with the same level of technology, $A_{J,-1}(i) = A_{J,-1}\ \forall i$, prior to R&D in the first period. As a result, they make identical decisions, and this symmetry is preserved in all future periods.[14,15]

There is a unit mass of R&D inputs, yielding

$$R_{N,t} + R_{E,t} = 1\ \forall t. \tag{9}$$

Thus, $R_{J,t}$ can be interpreted as the total share of R&D inputs used to improve technology J at time t. There is free mobility of R&D inputs between firms and technologies. The fixed set of research inputs is a stand-in for two offsetting forces, an increase in aggregate research inputs and an increase in the cost of generating a given technology growth rate (Bloom et al., 2020). This approach is consistent with existing literature on DTC and the environment (e.g., Acemoglu et al., 2012; Fried, 2018).[16]

Capital goods depreciate fully when they are used in the production of the final good. In the quantitative application, each period will be ten years. The investment price is fixed at one, implying that one unit of saved output can always be used to create one unit of any capital good. This yields the market clearing condition

$$\int_0^1 X_t(i)di \leq K_t, \tag{10}$$

where K_t is aggregate capital (i.e., output saved in period $t-1$).

[14] See Appendix Sections B.2 and B.3 for further discussion.

[15] In a competitive equilibrium, the set-up presented here is isomorphic to one where firms are infinitely lived and R&D productivity is given by $A_{J,t}(i) = \left(1 + \eta_J R_{J,t}(i)^{1-\lambda}\right) A_{J,t-1}$. This would open up the possibility of technological regress at the firm level, though it would not occur in equilibrium.

[16] See Appendix Section E.1 for an extension of the model that incorporates insights from 'second-wave' endogenous growth theory (e.g., Peretto, 1998) to eliminate scale effects.

3.1.5 Representative Household

The consumer side of the problem is standard. The representative household chooses a path of consumption $\{C_t\}_{t=0}^{\infty}$ to maximize lifetime utility

$$U \equiv \sum_{t=0}^{\infty} \beta^t L_t \frac{\tilde{c}_t^{1-\sigma} - 1}{1-\sigma}, \tag{11}$$

where $\tilde{c}_t = C_t/L_t$, and population growth is given by

$$L_{t+1} = (1+n)L_t. \tag{12}$$

The representative household owns both the capital stock and the R&D inputs, and it also receives all profits from the monopolists. The government ensures zero net revenue using lump-sum taxes or transfers. The budget constraint for the representative household is given by

$$C_t + K_{t+1} = L_t w_t + r_t K_t + \Pi_t + p_t^R + T_t, \tag{13}$$

where w_t is the wage, r_t is the rental rate of capital, Π_t is total profits of the capital good producers, p_t^R is total income earned by R&D inputs, and T_t is net government revenue from taxes and subsidies. The representative household takes factor prices, firm profits, technology, and policy as given. Since aggregate income is equal to final output, constraint (13) is also the market clearing condition for the final good.

The assumption of full depreciation ensures that the representative household will never save capital that goes unused in the next period, implying that the market clearing condition for capital (10) holds with equality for $t \geq 1$. The initial value K_0 is given, and I assume that it is low enough to ensure that the condition also holds at $t = 0$.

3.2 Analysis

As demonstrated in Appendix Section B.1, the first order conditions for the final good producer yield the following inverse demand function for capital good i:

$$p_{X,t}(i) \quad = \quad \alpha \left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right] \left(A_{N,t}(i) L_t\right)^{1-\alpha} X_t(i)^{\alpha-1}, \tag{14}$$

where $(\tau_t - 1) \geq 0$ is a value-added tax on energy. The intuition for the result is straightforward. The final good producer demands capital goods until marginal revenue is equal to marginal cost. Unlike the usual endogenous growth model, marginal revenue is equal to marginal product minus the cost of energy needed to operate capital goods.

The monopolists determine the production quantity of each capital good and the allocation of R&D inputs. See Appendix Section B.2 for a complete derivation of monopolist behavior. Given the iso-elastic inverse demand function, they set price equal to a constant markup over unit cost. Since capital must be rented from consumers, the unit cost is given by (1-τ_t^K)r_t, where τ_t^K is a subsidy for capital good purchases. For all subsequent analyses, I assume that $\tau_t^K = (1-\alpha)\ \forall t$, which undoes the monopoly distortion. For each i, capital producers choose the production quantity

$$X_t(i) = \alpha^{\frac{1}{1-\alpha}} r_t^{\frac{-1}{1-\alpha}} A_{N,t}(i) L_t \left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]^{\frac{1}{1-\alpha}}. \tag{15}$$

Applying these results, profits can be written as

$$\tilde{\pi}_{X,t}(i) = \tilde{\alpha} r_t^{\frac{-\alpha}{1-\alpha}} A_{N,t}(i) L_t \left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]^{\frac{1}{1-\alpha}} - (1-\eta_t^S) p_{E,t}^R R_{E,t}(i) - p_{N,t}^R R_{N,t}(i), \tag{16}$$

where $p_{J,t}^R(i)$ is the rent paid to R&D inputs used by firm i to improve technology J at time t, $\eta_t^S \in [0,1)$ is a subsidy for energy efficiency R&D, and $\tilde{\alpha} \equiv (1-\alpha)\alpha^{\frac{1}{1-\alpha}}$ is a constant.

Firms hire R&D inputs, $R_{N,t}(i)$ and $R_{E,t}(i)$, to maximize (16) subject to the R&D production function (8). The first order conditions yield the research arbitrage equation

$$\frac{(1-\eta_t^S) p_{E,t}^R(i)}{p_{N,t}^R(i)} = (1-\alpha)^{-1} \left(\frac{\frac{\tau_t p_{E,t}}{A_{E,t}(i)}}{1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}} \right) \cdot \left(\frac{A_{N,t}(i)}{A_{E,t}(i)} \right) \cdot \left(\frac{\eta_E R_{E,t}(i)^{-\lambda} A_{E,t-1}}{\eta_N R_{N,t}(i)^{-\lambda} A_{N,t-1}} \right). \tag{17}$$

The left-hand side gives the relative cost of using R&D inputs to improve the two types of technology. In equilibrium, this is equal to the relative benefit of hiring R&D inputs, which is given on the right-hand side.

It is more informative to interpret this research arbitrage condition in terms of aggregate variables. All capital good producers face an identical problem and make identical decisions (see Appendix Sections B.2 and B.3 for further discussion). Since there is a unit mass of capital good producers, $R_{J,t}(i) = R_{J,t}$ and $A_{J,t}(i) = A_{J,t}\ \forall i, J, t$. Market clearing implies that $X_t(i) = K_t$ and $E_t(i) = E_t\ \forall i, t$. Since there is free mobility for R&D inputs across firms, $p_{J,t}^R(i) = p_{J,t}^R\ \forall i, J, t$. In addition, due to Leontief production, $\frac{\tau_t p_{E,t}}{A_{E,t}}$ is the tax-inclusive the cost of energy per unit of final good production, and $1 - \frac{\tau_t p_{E,t}}{A_{E,t}}$ is the cost of non-energy inputs per unit of the final good.[17] Putting these results together, it is possible to rewrite research arbitrage condition (17) as

$$\frac{(1-\eta_t^S)p_{E,t}^R}{p_{N,t}^R} = (1-\alpha)^{-1}\left(\frac{\tau_t p_{E,t} A_{E,t}^{-1}}{\hat{\alpha}^{-1} w_t^{1-\alpha} r_t^{\alpha} A_{N,t}^{\alpha-1}}\right)\cdot\left(\frac{A_{N,t}}{A_{E,t}}\right)\cdot\left(\frac{\eta_E R_{E,t}^{-\lambda} A_{E,t-1}}{\eta_N R_{N,t}^{-\lambda} A_{N,t-1}}\right), \tag{18}$$

where $\hat{\alpha} = (1-\alpha)^{1-\alpha}\alpha^{\alpha}$. While the set-up of the Leontief DTC model presented here is different from the standard symmetric-sectors approach (e.g., Acemoglu, 2002; Acemoglu et al., 2012), the key forces are similar. The first fraction on the right-hand side is an *input price effect*. The numerator is the cost of purchasing enough energy to produce one unit of output, and the denominator is the cost of purchasing enough of the capital-labor composite to produce one unit of output. The second fraction highlights the role of the low elasticity of substitution. Increases in $A_{N,t}$ raise the return to investing in $A_{E,t}$ and vice versa. This is related to the market size effect in more standard models. With Leontief production, relative factor supply is inversely related to relative factor productivity. The final term is a *research productivity effect*. The return to investing in a particular type of R&D is increasing in the efficiency of the research process. Research efficiency depends on inherent productivity, η_J, accumulated knowledge, $A_{J,t-1}$, and the impact of decreasing returns, $R_{J,t}^{-\lambda}$.

With free mobility, the rental rate of R&D inputs must be the same for both technologies: $p_{N,t}^R = p_{E,t}^R = p_t^R$. For intuition and the calibration, it is helpful to think about R&D incentives in terms of energy expenditure as a fraction of final output.[18]

Definition 1. *The* energy expenditure share *($\theta_{E,t}$) is tax-inclusive energy expenditure as a fraction of final output, i.e.,* $\theta_{E,t} \equiv \frac{\tau_t p_{E,t} E_t}{Y_t}$.

In the absence of environmental policy,

$$\theta_{E,t} = \frac{p_{E,t}/A_{E,t}}{1 - p_{E,t}/A_{E,t}}.$$

In this case, it is possible to rewrite research arbitrage equation (18) to show the equilibrium

[17] See Appendix Section B.4 for derivations of factor shares and factor prices.

[18] In the model, payments to energy suppliers are not a source of income for the representative household, because there is no value added in the energy industry. The definition of energy expenditure used here corresponds to the data shown in Figure 2, where the energy share is also calculated as aggregate, tax-inclusive expenditure on final-use energy divided by GDP. Since final-use energy is an intermediate input, payments to the final energy suppliers are also excluded from GDP in the data.

relationship between technology growth rates, factor shares, and R&D allocations:

$$\frac{1+g_{A_E,t}}{1+g_{A_N,t}} = \left(\frac{\theta_{E,t}}{1-\alpha}\right) \cdot \left(\frac{\eta_E R_{E,t}^{-\lambda}}{\eta_N R_{N,t}^{-\lambda}}\right). \tag{19}$$

After aggregating across capital good producers, equation (7) implies that the elasticity of final output with respect to energy efficiency is given by $\frac{\partial \ln Y_t}{\partial \ln A_{E,t}} = \frac{p_{E,t}/A_{E,t}}{1-p_{E,t}/A_{E,t}} = \theta_{E,t}$, and the elasticity with respect to non-energy technology is given by $\frac{\partial \ln Y_t}{\partial \ln A_{N,t}} = 1-\alpha$. These elasticities play a significant role in determining the final good producer's demand for capital goods and the technologies they embody, which in turn affects the profits of capital good producers.[19]

Importantly, there is an asymmetry in the way that technological progress affects these elasticities. Changes in $A_{N,t}$ have no effect on $\frac{\partial \ln Y_t}{\partial \ln A_{N,t}}$, but changes in $A_{E,t}$ have a negative effect on $\frac{\partial \ln Y_t}{\partial \ln A_{E,t}}$. In the model, $p_{E,t}$ increases every period, driving up the relative benefit of increasing $A_{E,t}$ – as captured by the energy expenditure share – and prompting capital good producers to allocate R&D inputs into energy efficiency, which drives the energy expenditure share back down. On a BGP, when technology growth rates and R&D allocations are constant, the energy expenditure share must also be constant, implying that the increases in $A_{E,t}$ are just large enough to offset the increases in energy prices.

So far, all of the expressions for R&D allocations have been presented in terms of endogenous variables. Appendix Section B.5 uses equation (17) and free mobility of R&D inputs to show that the equilibrium R&D allocations are given by

$$R_{E,t} = \Gamma\left(\frac{\tau_t p_{E,t}}{A_{E,t-1}}\right), \tag{20}$$

$$R_{N,t} = 1 - R_{E,t}, \tag{21}$$

where $\Gamma(\frac{\tau_t p_{E,t}}{A_{E,t-1}})$ is some well-defined function with $\Gamma'(\cdot) > 0$. This result shows that the intuition for balanced growth holds in the presence of environmental policy. On a balanced growth path, $\frac{\tau_t p_{E,t}}{A_{E,t-1}}$ must be constant, implying that energy efficiency and tax-inclusive energy prices grow at the same rate.

[19] Hart (2013) and Hassler et al. (2021a) identify similar relationships between equilibrium growth rates and the energy expenditure share. As noted by Hassler et al. (2021a), this result is related to the Uzawa (1961) steady state growth theorem. Energy, like capital, can be produced from output. In the absence of taxation, $A_{E,t}/p_{E,t}$ can be thought of as the effective energy-augmenting technology. The steady state theorem says that this technology must be constant on a BGP. Focusing on capital, Acemoglu (2003) shows how endogenous innovation ensures that an economy will endogenously satisfy the Uzawa condition when the elasticity of substitution between factors is less than one.

Utility maximization yields the Euler equation,

$$\left(\frac{\tilde{c}_t}{\tilde{c}_{t+1}}\right)^{-\sigma} = \beta r_{t+1}, \tag{22}$$

and the terminal condition,

$$\lim_{T\to\infty} \beta^T \tilde{c}_T^{-\sigma} K_{T+1} = 0. \tag{23}$$

As shown in Appendix Section B.4, the real interest rate, which is equal to the rental rate of capital, is given by

$$r_t \;=\; \alpha \left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}}\right] (A_{N,t} L_t)^{1-\alpha} K_t^{\alpha-1}. \tag{24}$$

3.3 Equilibrium

Noting that all capital good producers make identical decisions, the definition of a competitive equilibrium can be expressed in terms of aggregate variables.

Definition 2. *A* competitive equilibrium *is a sequence of prices, $\{w_t, p_{X,t}, r_t, p_t^R, p_{E,t}\}_{t=0}^{\infty}$, allocations, $\{C_t, K_t, L_t, E_t, R_{N,t}, R_{E,t}\}_{t=0}^{\infty}$, technology levels, $\{A_{N,t}, A_{E,t}\}_{t=0}^{\infty}$, and environmental policies, $\{\tau_t, \eta_t^S\}_{t=0}^{\infty}$, such that each of the following conditions holds $\forall t$: (i) the economy obeys market clearing conditions for final goods (13); (ii) R&D allocations solve (20) and (21); (iii) the dynamics for technology follow (5) and (8); (iv) consumer behavior follows the Euler equation (22) and terminal condition (23); (v) factor prices are given by (4), (14), (24), (B.7), and (B.34); (vi) the economy obeys laws of motion for total extracted energy (6) and population (12); and (vii) initial conditions $A_{J,-1}$ for $J = E, N, V$, as well as K_0, L_0, and $\bar{E}_{-1}$, are given.*

3.4 Balanced Growth under Laissez-Faire

In this section, I examine the balanced growth path in a laissez-faire equilibrium (LFBGP). I show that the LFBGP is consistent with the stylized facts regarding energy use, prices, and efficiency presented in Sections 2.2 and 2.3, as well as the standard balanced growth facts used to discipline growth models. I focus on the relevant intuition. The complete analysis is presented in Appendix Section B.7.1.[20] I use asterisks ($*$) to denote BGP values and g_Z to denote the growth rate of any variable Z.

[20] Appendix Section B.6 derives the dynamical system.

Definition 3. *A* laissez-faire equilibrium *is a* competitive equilibrium *without environmental policy, i.e.,* $\tau_t = 1$ *and* $\eta_t^S = 0\ \forall t$.

Definition 4. *A* balanced growth path (BGP) *is a path along which macroeconomic aggregates and technologies,* $\{Y_t, K_t, C_t, E_t, A_{E,t}, A_{N,t}\}$*, all grow at constant rates.*

Definition 5. *A* BGP *is* unique *if the growth rates of the macroeconomic aggregates and technologies do not depend on the initial conditions.*[21]

Given that energy extraction costs depend on cumulative extraction, a LFBGP can only exist asymptotically, unless the economy happens to start on a LFBGP. To characterize the LFBGP, it is necessary to place some restrictions on the exogenous parameters. The parameter g_{A_V} controls growth in the the exogenous component energy prices. When $g_{A_V} > 0$, technological progress in the extraction sector pushes down energy prices, and the opposite occurs when $g_{A_V} < 0$. I assume that

$$(1 - g_{A_V}) \in \left(\frac{1}{[(1+n)(1+\eta_N)]^{\psi}}, \frac{(1+\eta_E)^{1+\psi}}{(1+n)^{\psi}} \right). \tag{A1}$$

The lower bound rules out the case where energy prices decrease even when there is no investment in energy efficiency. The upper bound rules out the case where improvements in energy efficiency cannot keep pace with rising energy prices, even though all R&D is devoted to improving energy efficiency.

Figure 4 shows how the LFBGP growth rates of technology, $g^*_{A_N}$ and $g^*_{A_E}$, are determined. I start by deriving (RD-BGP). This is a relationship between the two growth rates that must hold on any LFBGP with increasing flow energy use ($g^*_E > 0$). On any BGP, R&D allocations must remain fixed. From the expressions for R&D allocations in (20) and (21), $\frac{p_{E,t}}{A_{E,t-1}}$ is constant on a LFBGP. Intuitively, this occurs because of the non-linear relationship between energy efficiency, $A_{E,t}$, and the cost of energy per unit of output, $\frac{p_{E,t}}{A_{E,t}}$. When energy prices increase, monopolists have greater incentive to invest in energy efficient technology, but this incentive dissipates as energy efficiency improves and the cost of energy per unit of output falls. As a result, energy prices and energy efficiency grow at the same constant rate, $g^*_P = g^*_{A_E}$, on a LFBGP. As discussed in the context of (19), this implies that the energy expenditure share, $\theta_{E,t} = \frac{p_{E,t}/A_{E,t}}{1-p_{E,t}/A_{E,t}}$, is constant on a LFBGP, matching the U.S. data.

[21] As in a standard neoclassical growth model, this definition of uniqueness allows the BGP levels of the endogenous variables to depend on the initial level of technology. Section 5.2.1 shows that the LFBGP levels of the endogenous variables are affected by the initial level of energy efficiency.

Definition 6. Total factor productivity *is defined as in the standard neoclassical growth model, i.e.,* $TFP_t \equiv \frac{Y_t}{K_t^\alpha L_t^{1-\alpha}}$.

It is immediate that

$$TFP_t = A_{N,t}^{1-\alpha}\left[1 - \frac{p_{E,t}}{A_{E,t}}\right]. \tag{25}$$

Since $\frac{p_{E,t}}{A_{E,t}}$ is constant on the LFBGP, TFP grows at factor $(1+g_{A_N}^*)^{1-\alpha}$, which is also constant. Since the consumer problem and capital accumulation equation are standard, gross output, final output, consumption, and the capital stock will all grow at factor $(1+g_{A_N}^*)(1+n)$ as in the standard neoclassical growth model.

Given the Leontief production function (2), $E_t = Q_t/A_{E,t}$ and the growth factor of flow energy use is

$$1 + g_E^* = \frac{1+g_{A_N}^*}{1+g_{A_E}^*}(1+n). \tag{26}$$

On a LFBGP with increasing energy use, cumulative energy grows at the same rate as flow energy use. Combined with the fact that $g_{A_E}^* = g_p^*$ and the expression for energy extraction costs (4), this gives

$$1 + g_{A_E}^* = (1 - g_{A_V})(1+g_E^*)^\psi. \tag{27}$$

Combining (26) and (27) yields

$$g_{A_E}^* = (1-g_{A_V})^{\frac{1}{1+\psi}}\left[(1+g_{A_N}^*)(1+n)\right]^{\frac{\psi}{1+\psi}} - 1. \tag{RD-BGP}$$

This key equation describes an endogenous relationship between $g_{A_E}^*$ and $g_{A_N}^*$ that must hold on any LFBGP with increasing energy use. It describes an upward sloping curve in (g_{A_N}, g_{A_E}) space. Faster growth in non-energy technology (higher $g_{A_N}^*$) yields faster growth in energy prices (higher g_P^*), which implies faster growth in energy efficiency (higher $g_{A_E}^*$) on the LFBGP. The R&D market clearing condition (RD-MC) gives a downward sloping curve in (g_{A_N}, g_{A_E}) space,

$$g_{A_E}^* = \eta_E\left[1 - \left(\frac{g_{A_N}^*}{\eta_N}\right)^{\frac{1}{1-\lambda}}\right]^{1-\lambda}, \quad g_{A_N}^* \in [0, \eta_N]. \tag{RD-MC}$$

Conditional on increasing flow energy use, (RD-BGP) and (RD-MC) uniquely determine the steady state growth rates of technology as shown in Figure 4.

I now discuss an additional assumption on g_{A_V}, which ensures that the LFBGP has

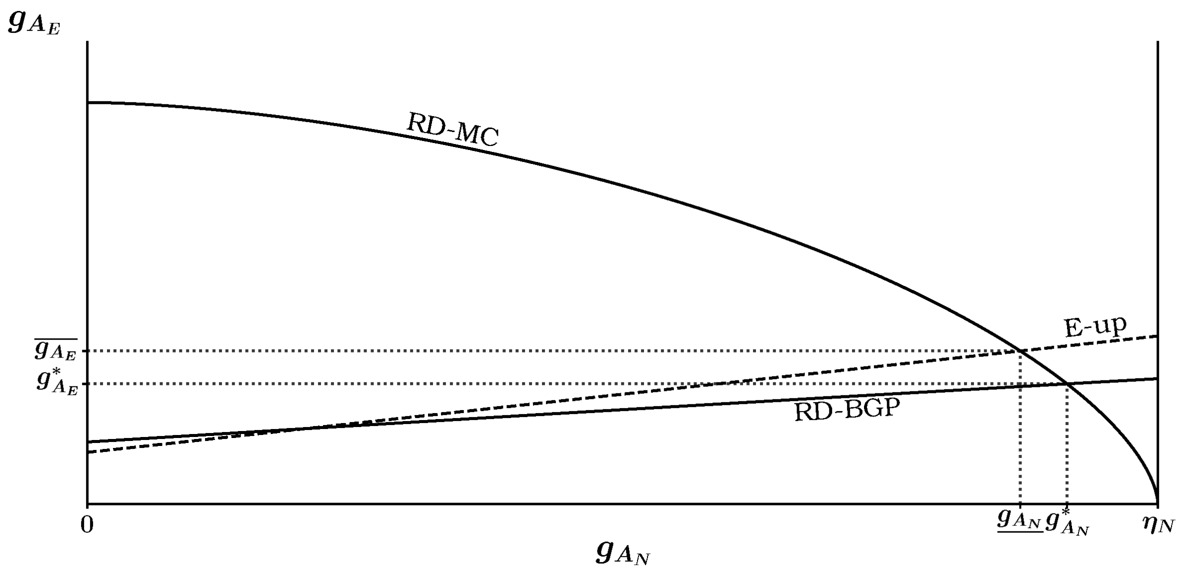


Figure 4: LFBGP technology growth rates. The intersection of (RD-BGP) and (RD-MC) gives the unique growth rates, $(g^*_{A_N}, g^*_{A_E})$, for a LFBGP with increasing energy use. The intersection of (RD-MC) and (E-up) defines the cutoff values $(\underline{g_{A_N}}, \overline{g_{A_E}})$ for increasing energy use. The LFBGP has increasing energy use if and only if $g^*_{A_N} > \underline{g_{A_N}}$ (equivalently, $g^*_{A_E} < \overline{g_{A_E}}$).

increasing energy use. To determine which pairs of technology growth rates lead to increasing energy use on the LFBGP, consider the equation

$$g^*_{A_E} = (1 + g^*_{A_N})(1 + n) - 1, \tag{E-up}$$

which gives all of the pairs for which $g^*_E = 0$ (i.e., all the pairs for which Q_t and $A_{E,t}$ have the same BGP growth rate). Define $(\underline{g_{A_N}}, \overline{g_{A_E}})$ as the solution to (E-up) and (RD-MC). This is the point where all R&D inputs are being used, and the growth rate of energy use is zero. Again, this point is shown on Figure 4. While there is no explicit algebraic solution to these equations, they determine $\underline{g_{A_N}}$ and $\overline{g_{A_E}}$ in terms of exogenous parameters. The LFBGP growth rate of energy use is positive if and only if $g^*_{A_N} > \underline{g_{A_N}}$ (equivalently, $g^*_{A_E} < \overline{g_{A_E}}$). A necessary and sufficient condition for this to hold is that

$$1 - g_{A_V} < (1 + \underline{g_{A_N}})(1 + n). \tag{A2}$$

Geometrically this implies that (RD-BGP) lies below (E-up) at $\underline{g_{A_N}}$, which in turn implies that $g^*_{A_N} > \underline{g_{A_N}}$. Thus, energy use is indeed increasing on the LFBGP as assumed above. Assumption (A2) also rules out a LFBGP with constant or decreasing energy use, implying that the LFBGP is unique.[22]

It is straightforward to characterize the rest of the LFBGP. As noted above, $g^*_{A_N}$ gives the growth rate of $TFP_t^{\frac{1}{1-\alpha}}$ on the LFBGP, which is also the per capita growth rate of consumption, capital, and output. In addition, the expressions for R&D allocations in (20) and (B.37) pin down the energy expenditure share once $g^*_{A_E}$ is known. Proposition 1 summarizes and extends the results from this section.

Proposition 1. *Let Assumptions (A1) and (A2) hold. For a sufficiently low β, there exists a* unique BGP *in a* laissez-faire equilibrium*. On this BGP, (i) R&D allocations are implicitly given by*

$$R^*_E = \left\{ \frac{\left[(1 + \eta_N (1 - R^*_E)^{1-\lambda})^{\frac{\alpha}{1-\alpha}} (1+n)(1 - g_{A_V})^{\frac{1}{\psi}} \right]^{\frac{\psi}{1+\psi}} - 1}{\eta_E} \right\}^{\frac{1}{1-\lambda}} \quad \textit{and } R^*_N = 1 - R^*_E;$$

(ii) output, consumption, and the capital stock grow at the same constant factor, $(1+g^*_{A_N})(1+n)$; (iii) the real interest rate is constant; (iv) flow energy use grows at constant factor $1 + g^*_E = \frac{1+g^*_{A_N}}{1+g^*_{A_E}}(1+n) > 0$; and (v) the expenditure shares of energy, capital, labor, R&D inputs and profits are all constant.

Proof. The intuition is provided above. See Appendix Section B.7.1 for the complete proof. □

This analysis shows that the LFBGP is consistent with the stylized facts observed in the U.S. data: the energy expenditure share is constant, energy prices and flow energy use are both increasing, and the energy intensity of output is decreasing. It also shows that the model recreates the usual balanced growth facts: income per capita grows at a constant rate, and the capital-output ratio, savings rate, rental rate, and labor share of income are all constant (see, e.g., Jones, 2016). However, this analysis does not say whether the economy will necessarily converge to the LFBGP. In Appendix Section B.7.2, I show numerically that the calibrated model is locally saddle-path stable for the baseline calibration and all of the robustness calibrations.

[22] If flow energy use grows at a constant or negative rate, the growth rate of the cumulative energy use eventually converges to zero. In this case, the growth of the energy price is determined entirely by its exogenous component, and (RD-BGP) would be replaced by $g^*_{A_E} = g^*_p = -g_{A_V}$. But, Assumption (A2) implies that this equation would lie below (RD-BGP) at $\underline{g_{A_N}}$ in Figure 4, implying that this updated equation would actually lead to increasing energy use, creating a contradiction.

3.5 Balanced Growth with Environmental Policy

In this section, I characterize a BGP with environmental policy (EPBGP) and analyze the impact of environmental policy on long-run outcomes. Again, I focus on the relevant intuition. The complete analysis is presented in Appendix Section B.8.1.

Definition 7. *An* equilibrium with environmental policy *is a* competitive equilibrium *with* $\tau_t = \tau_0(1+g_\tau)^t$ *and* $\eta_t^S = \eta^S$*, where* $g_\tau > 0$*,* $\eta^s \in [0,1)$*, and* $\tau_0 \geq 1$.[23]

For the remainder of the analysis, I assume that

$$(1+g_\tau) < \frac{1+\eta_E}{1-g_{A_V}}. \tag{A3}$$

If Assumption (A3) does not hold, improvements in energy efficiency cannot keep up with rising tax-inclusive energy prices, even when all R&D inputs are devoted to improving energy efficiency. In addition, the following condition will be important for determining whether the long-run growth rate of energy use is positive:

$$(1+g_\tau) \geq \frac{1+\underline{\underline{g_{A_E}}}}{1-g_{A_V}}. \tag{A4}$$

The steps for analyzing an EPBGP are similar to those for analyzing the LFBGP. The R&D market clearing condition (RD-MC) is unchanged by the introduction of environmental policy. Thus, the impacts of policy will depend on changes to the balanced growth condition for technology growth rates (RD-BGP). On an EPBGP, R&D allocation equations (20) and (21) now imply that the growth rate of energy efficiency is equal to growth rate of tax-inclusive energy prices, $g^*_{A_E} = (1+g_\tau)(1+g^*_P) - 1$. Combining this result with the law of motion for extraction costs (4) yields

$$1 + g^*_{A_E} = (1+g_\tau)(1-g_{A_V})(1+g^*_{\overline{E}})^\psi, \tag{28}$$

where $g_{\overline{E}_t} \equiv \frac{\bar{E}_t - \bar{E}_{t-1}}{\bar{E}_{t-1}}$ is the growth rate of cumulative energy use. Since energy efficiency grows faster than extraction costs, $\lim_{t\to\infty} \frac{p_{E,t}}{A_{E,t}} = 0$ and $\lim_{t\to\infty} \theta_{E,t} = \frac{\tau_t p_{E,t}}{A_{E,t}}$, which is constant.

There are two possible EPBGP outcomes. Similar to the LFBGP, each is only possible asymptotically. If flow energy use grows at a constant positive rate ($g^*_E > 0$), then in the

[23] I restrict the formal analysis to the case of exponentially increasing taxes and a fixed research subsidy for analytic convenience. I relax these assumptions in the quantitative analyses.

limit, cumulative energy use will grow at the same rate ($g^*_{\bar{E}} = g^*_E$). Otherwise, the growth rate of cumulative energy use converges to zero ($g^*_{\bar{E}} = 0$).

To start, I consider the case where energy use is increasing on the EPBGP, $g^*_E > 0$. Following the same steps as in the previous section, the fact the energy efficiency grows at the same rate as tax-inclusive energy prices implies

$$g^*_{A_E} = \left[(1+g_\tau)(1-g_{A_V})\right]^{\frac{1}{1+\psi}} \left[(1+g^*_{A_N})(1+n)\right]^{\frac{\psi}{1+\psi}} - 1, \qquad \text{(RD-BGP}')$$

which is identical to the LFBGP condition (RD-BGP) if $g_\tau = 0$. The intersection of market clearing condition (RD-MC) and the updated balanced growth condition (RD-BGP′) determines $g^*_{A_N}$ and $g^*_{A_E}$. The impact of energy taxes can be seen by comparing these results to those in Figure 4. If there is a positive growth rate of energy taxes, $g_\tau > 0$, then (RD-BGP′) lies strictly above (RD-BGP). As long as the intersection of (RD-BGP′) and (RD-MC) occurs to the right of $\underline{g_{A_N}}$, then the EPBGP will have increasing energy use. This will be true if and only if Assumption (A4) does not hold. As in the laissez-faire case, all of the macroeconomic variables grow at factor $(1+g^*_{A_N})(1+n)$ on the EPBGP. Compared to the LFBGP, the positive growth rate of energy taxes decreases $g^*_{A_N}$ and increases $g^*_{A_E}$, implying that the BGP growth rates of energy use and all macroeconomic aggregates fall. The equilibrium growth rates do not depend on either the level of the energy taxes or the existence of R&D subsidies.

Now, I consider the case where $g^*_E \leq 0$. Figure 5 shows how $g^*_{A_N}$ and $g^*_{A_E}$ are determined in the case. With zero growth in cumulative energy use, $\bar{g}^*_E = 0$, tax-inclusive energy prices are driven entirely by exogenous forces:

$$g^*_{A_E} = (1+g_\tau)(1-g_{A_V}) - 1. \qquad \text{(RD-BGP}'')$$

The equilibrium technology growth rates in this case are given by the intersection of the market clearing condition (RD-MC) and the updated balanced growth condition (RD-BGP″). The results are shown in Figure 5. Condition (RD-BGP″) is a horizontal line is (g_{A_N}, g_{A_E}) space. Assumptions (A1)–(A3) guarantee that the two curves intersect. For this result to be the endogenous outcome of the model, energy use cannot be increasing on the EPBGP, i.e., $g^*_{A_N} < \underline{g_{A_N}}$. This holds true if and only if Assumption (A4) holds. The assumption says that (RD-BGP″) lies above (E-up) at $\underline{g_{A_N}}$. Once again, increases in g_τ slow the growth rate of flow energy use and the other macroeconomic aggregates, while subsidies and the level of taxation have no effect on long-run growth rates.

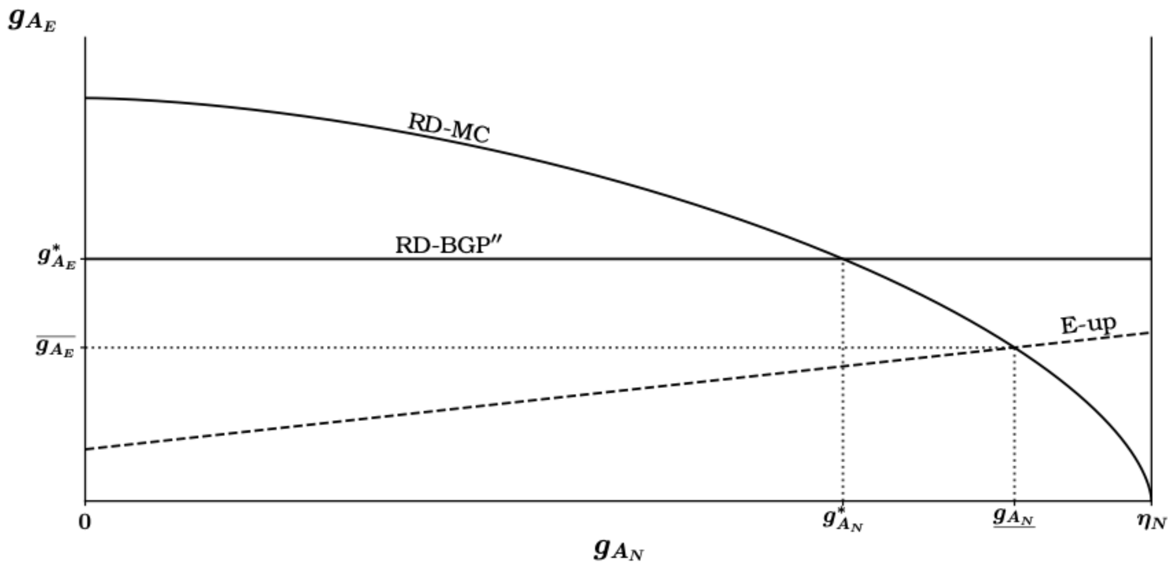


Figure 5: EPBGP technology growth rates. The intersection of (RD-BGP″) and (RD-MC) gives the unique growth rates, $g^*_{A_N}$ and $g^*_{A_E}$, for an EPBGP with decreasing flow energy use. The intersection of (RD-MC) and (E-up) defines the cutoff values, $\underline{g_{A_N}}$ and $\overline{g_{A_E}}$, that determine whether an EPBGP has a positive growth rate of flow energy use. The EPBGP has decreasing flow energy use if and only if $g^*_{A_N} < \underline{g_{A_N}}$ (equivalently, $g^*_{A_E} > \overline{g_{A_E}}$).

The EPBGP results are summarized and extended in Proposition 2.

Proposition 2. *Let Assumptions (A1)–(A3) hold. For a sufficiently low β, there exists a* unique BGP *in an* equilibrium with environmental policy. *Along the BGP, (i) if Assumption (A4) does not hold, R&D are implicitly given by*

$$R^*_E = \left\{ \frac{\left[(1+\eta_N(1-R^*_E)^{1-\lambda})^{\frac{\alpha}{1-\alpha}}(1+n)[(1+g_\tau)(1-g_{A_V})]^{1/\psi}\right]^{\frac{\psi}{1+\psi}} - 1}{\eta_E} \right\}^{\frac{1}{1-\lambda}} \quad \text{and } R^*_N = 1 - R^*_E,$$

but if Assumption (A4) does hold, R&D allocations are given by

$$R^*_E = \left[\frac{(1-g_{A_V})(1+g_\tau)-1}{\eta_E}\right]^{\frac{1}{1-\lambda}} \quad \text{and } R^*_N = 1 - R^*_E;$$

(ii) output, consumption, and the capital stock grow at the same constant factor $(1+g^*_{A_N})(1+n)$; (iii) the real interest rate is constant; (iv) if Assumption (A4) does not hold, flow energy use grows at factor $1+g^*_E = \frac{1+g^*_{A_N}}{1+g^*_{A_E}}(1+n) > 0$, but if Assumption (A4) does hold, $g^*_E \leq 0$; (v) the expenditure shares of energy, capital, labor, R&D inputs, and profits are all constant.

Proof. The intuition is provided above. See Appendix Section B.8.1 for the complete proof. □

The above results imply that neither the constant R&D subsidy, nor the level of energy taxes, has an impact on the BGP growth rates of energy use or any of the other macroeconomic aggregates. These results hold because energy and non-energy inputs are complements in the production of the final good. Given the Leontief production function (2), $g_{E,t}$ depends positively on $g_{A_N,t}$ and negatively on $g_{A_E,t}$. To decrease g^*_E, policy interventions must permanently reallocate R&D resources towards energy efficiency. In other words, policy must increase $g^*_{A_E}$. As demonstrated by the expression for R&D allocations (20), this will only occur if policy permanently increases the growth rate of tax-inclusive energy prices. Neither the R&D subsidies nor the level of energy taxation has this effect.

Since they affect incentives for R&D, however, subsidies and the level of taxes will have an impact on the EPBGP levels of energy use and the other macroeconomic aggregates, including consumption. They will also affect the growth rate of energy use and the other macroeconomic aggregates on the transition path. When a constant subsidy for energy efficiency R&D is introduced, energy efficiency grows quickly, reducing energy use. The rapid growth in energy efficiency and reduced growth in energy extraction costs both push down the energy expenditure share, decreasing future incentives for energy efficiency R&D. Eventually, the energy expenditure share falls far enough to fully offset the subsidy, and the technology growth rates return to their original levels. The same reasoning applies to the level of energy taxes. In Sections 5.1.2 and 5.2.2, I use the quantitative model to investigate the role of subsidies along the transition path, focusing on a wider set of macroeconomic outcomes.

The following corollary collects and highlights some important implications from the proposition. These will be discussed further in the quantitative analysis.

Corollary 1. *In an* equilibrium with environmental policy*, the constant research subsidies* (η^S) *and the level of energy taxes* (τ_0) *have no effect on the BGP growth rates of technology, energy use, or the other macroeconomic aggregates. The growth rate of energy efficiency* ($g^*_{A_E}$) *is increasing in the growth rate of energy taxes* (g_τ)*. The common growth rate of output, consumption, and capital per person* ($g^*_{A_N}$) *is decreasing in* g_τ*. The growth rate of energy use* (g^*_E) *is also decreasing in* g_τ*. In addition, if* g_τ *is high enough that Assumption (A4) holds with strict inequality, then flow energy use converges to zero in the limit. If Assumption (A4) holds with equality, then flow energy use is constant in the limit. If Assumption (A4) does not hold, then the growth rate of flow energy use is positive in the limit.*

3.6 Least-Cost Path to Achieve an Environmental Target

In this section, I examine the allocations and policies that maximize lifetime utility of the representative household (11), subject to an environmental constraint of the form $\sum_{t=0}^{T} E_t \leq E^{\text{target}}$. I refer to resulting allocations as the least-cost path and the difference between cumulative energy use and E^{target} as the remaining 'energy budget'. Intuitively, the constraint captures the case where policy is designed to stay within limits derived from climate science. Existing analyses suggest that, in order to meet targets highlighted in international agreements, economies will need to reduce final-use energy consumption, even in the presence of large-scale substitution from dirty to clean sources of primary energy (e.g., Williams et al., 2014; Rogelj et al., 2018). Section 5.1 examines this setting quantitatively, focusing on targets derived from the Paris Agreement.

The full theoretical analysis is presented in Appendix Section B.8.2. It considers the case of log preferences, which will also be used in the quantitative analysis. Here, I focus on the main results and intuition. I am particularly interested in comparing the roles of energy taxes and directed R&D policy in implementing the least-cost path. The literature on substitution between clean and dirty sources of energy has paid a great deal of attention to this question, focusing on whether subsidies for clean energy R&D are necessary to implement optimal allocations (e.g., Golosov et al., 2014; Acemoglu et al., 2012, 2016; Hart, 2019).

To determine the policy mix that will implement the least-cost path, I first solve the social planner problem to find the least-cost path allocations and then compare the results to the competitive equilibrium. I assume that the government has access to lump-sum taxes and transfers and can subsidize the production of capital goods. In addition, the government has access to two environmental policies: per-unit energy taxes (τ_t^u) and per-unit subsidies/taxes for energy efficiency R&D (η_t^U).[24] A positive value for η_t^U indicates a subsidy. Given that the total quantity of R&D inputs is fixed, policy only affects the allocation of R&D inputs between the two types of technology, and a tax on energy efficiency R&D is equivalent to subsidy for non-energy R&D. The key distinction for R&D policy is which technology is favored by R&D policy, not whether the outcome is achieved with taxes or subsidies on R&D.

There are three types of externalities. First, energy extraction in period t increases the cost of energy extraction in future periods and uses up the energy budget. Second, R&D in period t affects research productivity in future periods. Third, there is a monopoly

[24] To facilitate the discussion of the BGP, the analysis so far has focused on the case of value-added taxes and subsidies for energy efficiency R&D. The competitive equilibrium with per-unit taxes and subsidies is the same with $\tau_t^u + p_{E,t} = \tau_t p_{E,t}$ and $p_t^R(1 - \eta_t^S) = p_t^R - \eta_t^u$.

distortion in the production of capital goods. The three policy instruments described above are sufficient to correct these three market failures and implement the least-cost path.

Proposition 3. *The government can implement the least-cost path using three policies: (1) a subsidy for capital good production* $\tau_t^K = (1-\alpha)\ \forall t$*, (2) a tax* ($\tau_t^u > 0\ \forall t$) *on energy use, and (3) a subsidy/tax* (η_t^u) *for energy efficiency R&D. In addition,* η_t^u *has the same sign as* $(g_{A_E,t+1} - g_{A_E,t})$*. In other words, the government subsidizes energy efficiency R&D in period* t *if energy efficiency grows faster in period* $t+1$ *than in period* t*, taxes energy efficiency R&D if the growth rate slows between* t *and* $t+1$*, and does neither if the growth rate is constant between periods.*

Proof. See Appendix Section B.8.2. □

As stressed in Golosov et al. (2014), separate instruments correct separate market failures. The tax forces the final good producer to internalize all of the external costs of energy use. The capital good subsidy undoes the monopoly distortion, and the R&D subsidy/tax forces capital good producers to internalize the intertemporal knowledge spillovers. Acemoglu et al. (2012) arrive at a similar result in the context of substitution between clean and dirty sources of energy. To see the separate roles played by energy taxes and R&D taxes/subsidies, it is helpful to consider the extreme case where there is no externality from energy use.

Corollary 2. *Consider the case where* $\psi = 0$ *(i.e., exogenous energy prices) and* $\bar{E}^{target} = \infty$*. The government can implement the least-cost path using two policies: (1) a subsidy for capital good production* $\tau_t^K = (1-\alpha)\ \forall t$ *and (2) a subsidy/tax* (η_t^u) *for energy efficiency R&D, where* η_t^u *has the same sign as* $(g_{A_E,t+1} - g_{A_E,t})$*.*

Removing the externalities associated with energy use affects the role of energy taxes in implementing the least-cost path, but has no effect on the role of R&D taxes/subsidies, which only address intertemporal spillovers. Intuitively, policy should favor whichever technology has a bigger gap between the social and private return to R&D. Since R&D allocations are interior along the least-cost path, the social return to R&D must be equalized across technologies, and policy should favor the technology with larger external returns and smaller private returns. When $g_{A_E,t+1} > g_{A_E,t}$ and $g_{A_N,t+1} < g_{A_N,t}$ along the least-cost path, the intertemporal spillover from current R&D to future research productivity is more important for energy efficiency at time t. The opposite result holds when $g_{A_E,t+1} < g_{A_E,t}$ and $g_{A_N,t+1} > g_{A_N,t}$ along the least-cost path. Hart (2019) and Greaker et al. (2018) derive similar relationships between growth rates and R&D policy when studying substitution between clean and dirty sources of energy.

Table 1: Calibration Results

Parameter	Value	Description	Source
α	.35	Capital share of income	Golosov et al. (2014)
δ	1	Depreciation	Golosov et al. (2014)
β	0.86	Discount factor	Golosov et al. (2014)
σ	1	Inter-temporal substitution	Golosov et al. (2014)
n	0.11	Population growth	EIA
λ	0.40	Research dim. returns	Calibrated
η_E	0.84	Research efficiency	Calibrated
η_N	0.22	Research efficiency	Calibrated
ψ	1.26	Extraction cost convexity	Calibrated
$\bar{E}_{-1}/E_0$	15.7	Energy stock/flow	Calibrated
g_{A_V}	-0.16	Extraction technology growth	Calibrated

As stressed throughout this paper, transition dynamics play an important role in meeting environmental policy goals. Proposition 3 implies that directed R&D policy is necessary to implement the least-cost path whenever there is a transition path with changing technology growth rates. As shown in Section 5.1.2, the least-cost path in the quantitative application is implemented with R&D policy that favors non-energy technology, further highlighting the result that separate instruments correct separate market failures.

4 Calibration

This section presents the calibration. Table 1 summarizes the results.

4.1 External Parameters and Data

The period length is ten years. The consumer and non-energy production portions of the model are standard. I follow Golosov et al. (2014) and set $\alpha = 0.35$, $\delta = 1$ (full depreciation), $\sigma = 1$, and $\beta = 0.86$. I assume that the economy starts without environmental policy. All taxes and subsidies, therefore, are relative to 'business as usual' case, which serves as the baseline.

Data sources and details can be found in Appendix A. Due to limitations on energy expenditure data, I restrict attention to the period 1971-2016. Over this period, the average growth rate of final output in the United States was $g_Y = 0.33$ (2.9%/year). Population growth was $n = 0.11$ (1.0%/year). The growth rate of income per capita was $g^*_{A_N} = 0.20$ (1.9%/year). In addition, final-use energy consumption in the United States grew at rate $g^*_E = 0.06$ (0.6%/year). So, $g^*_{A_E} = 0.25$ (2.3%/year), which is also the LFBGP growth rate of energy prices, g_P^*. The average energy expenditure share in the data is 8.41%, and the expenditure share of R&D is 2.65%.[25]

4.2 R&D Calibration

The R&D production functions have three unknown parameters, η_N, η_E, and λ. Combining the law of motion for technology (8) and the implications of research arbitrage (19) gives

$$\frac{R_{E,t}}{1-R_{E,t}} = \frac{g_{A_E,t}}{1+g_{A_E,t}} \frac{1+g_{A_N,t}}{g_{A_N,t}} \frac{\theta_{E,t}}{1-\alpha}, \tag{29}$$

which implies that the LFBGP R&D allocations can be found without knowing any of the parameters of the R&D production function. Bringing this equation to the data yields $R_E^* = 0.13$ and $R_N^* = 0.87$.

As shown in Appendix Section B.4, the R&D share of income is

$$\theta_R^* = (1-\lambda) \cdot \alpha(1-\alpha) \cdot \frac{g_{A_N}^*/R_N^*}{1+g_{A_N}^*} \tag{30}$$

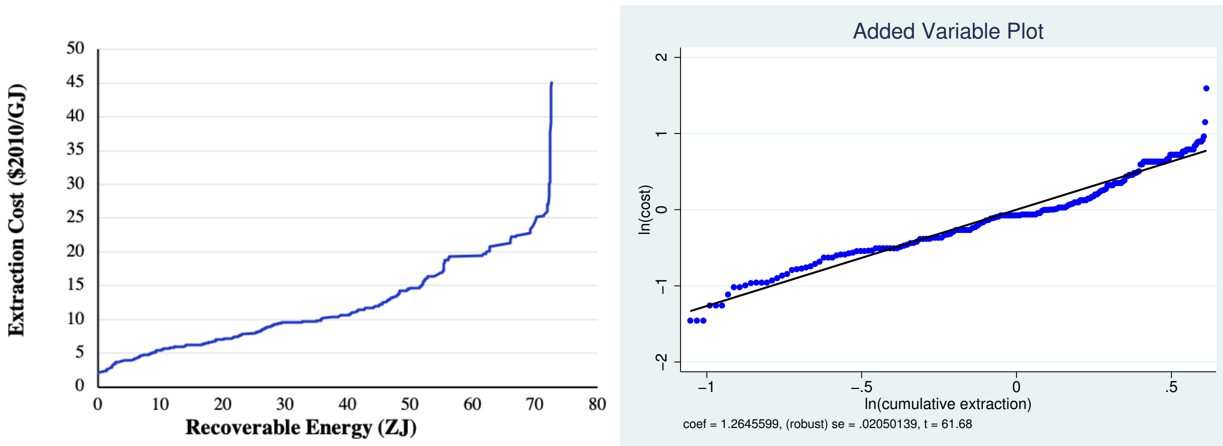
on a LFBGP, which yields $\lambda = 0.40$. With an estimate of λ, I use the law of motion for technology (8) to solve for $\eta_E = 0.84$ and $\eta_N = 0.22$. The results suggest that improving energy efficiency technology is inherently easier than improving non-energy technology.[26]

4.3 Energy Sector Calibration

On the LFBGP, both flow and cumulative energy use grow at the same constant rate. The growth rate of flow energy use, g_E^*, is observed in the data. Thus, I calculate the initial level of extracted energy as $E_0/\bar{E}_{-1} = g_E^*$, where E_0 is flow energy use in the first period and $\bar{E}_{-1}$ is the cumulative energy use prior to the first period. The calibration yields $\bar{E}_{-1}/E_0 = 15.7$.

[25] Due to data limitations, the R&D share is calculated from 1981-2016. In the data, the R&D share of output is 2.58%. Unlike the model, the national accounts data include R&D expenditure as a component of GDP. Adjusting for this difference gives an R&D share of 2.65%.

[26] Intuitively, this results follows from two facts observed in the data. First, the energy expenditure share is small compared to the capital share of income, suggesting that there is little incentive for firms to invest heavily in energy efficiency. Second, the growth rate of energy efficiency is higher than the growth rate of non-energy technology. If the relative benefit of investing in energy efficiency in low, but the growth rate is high, it must be the case that the cost of improving energy efficiency is relatively low. Fried (2018) examines DTC in clean versus dirty sources of energy, as well as non-energy technology, and finds diminishing returns of $\lambda = 0.21$. I use this value in robustness analyses. Existing empirical work in endogenous growth models finds cost functions that are approximately quadratic in research effort (Acemoglu et al., 2018; Akcigit and Kerr, 2018). This would give $\lambda = 0.50$.



(a) Extraction Cost Curve

(b) Estimating ψ

Figure 6: Extraction Costs. Panel (a) presents extraction cost curves in terms of final-use energy availability. Panel (b) shows the fit of (32) when estimated by OLS. Original cost and availability estimates are from McGlade and Ekins (2015b), and adjustments for the average efficiency at which primary energy is converted to final-use energy are calculated using data from Rogner et al. (2012), the IEA, and the EIA.

Conditional on ψ, the ratio between the cumulative stock and per period flow of energy use determines the degree to which energy prices respond to policy-induced changes in energy use.

The parameter ψ governs the shape of the energy extraction cost curve. To calibrate this parameter, I use estimates of energy availability and extraction costs from McGlade and Ekins (2015b). They estimate future global extraction cost curves for coal, oil, and natural gas. I combine these curves to calculate total final-use energy availability at each extraction cost. Figure 6 plots the the results. The variable on the vertical axis, which I will call $cost_m$, is the extraction cost measured in 2010 dollars. The variable on the horizontal axis, which I will call D_m, is the amount of final-use energy available at $cost_m$ or less in 2010. It is measured in zettajoules (ZJ). I divide the range of D_m into 200 equally-spaced grid points and find the extraction cost at each point. These grid points are the unit of observation, m, in the regression described below.[27]

Taking logs, the law of motion for energy prices (4) is given by

$$\ln p_{E,t} = \ln A_{V,t}^{-1} + \psi \ln \left(\bar{E}_{-1} + \sum_{\hat{t}=0}^{\hat{t}-1} E_{\hat{t}} \right). \tag{31}$$

Here, $p_{E,t}$ is the extraction cost at time t, which corresponds to $cost_m$ in the data. Costs in

[27]See Appendix Section C.1 for further details about the data and construction of the figure.

the data are measured at a single point in time. Also, $\sum_{\hat{t}=0}^{\hat{t}-1} E_{\hat{t}}$ is the amount of energy used between periods 0 and $t-1$. This corresponds to D_m in the data, which is the amount of energy that could be extracted at a given cost. Parameter $\bar{E}_{-1}$ is cumulative energy used prior to start of the model. The estimates from McGlade and Ekins (2015b) do not include past extraction. Rogner et al. (2012) estimate that cumulative extraction of coal, natural gas, oil prior to 2005 has been 16.5 ZJ,[28] which I take as given for the regression. Finally $A_{V,t}^{-1}$ is the state of extraction technology at the time that costs are measured, which is held constant in the data. Thus, I estimate

$$\ln cost_m = b_0 + b_1 \ln\left(16.5 + D_m\right), \tag{32}$$

by ordinary least squares. The coefficient of interest is b_1, which provides the estimate of ψ. The results of the regression are shown in panel (b) of Figure 6. I find $\hat{\psi} = 1.26$.[29]

With an estimate of ψ, it is now possible to calibrate g_{A_V}. Taking the log of (4) and comparing across periods, the BGP relationship becomes $\ln(1-g_{A_V}) = \ln(1+g_P^*) - \psi \ln(1+g_E^*)$. Differential technological progress captures the growth in energy prices that cannot be explained by the shape of the extraction cost curve. This yields $g_{A_V} = -0.16$ over ten years (-1.7%/year). Technological progress in the energy extraction sector is significantly slower than technological progress in final good production, though extraction technology is still improving over time.[30] Finally, $A_{V,0}$ is a scale parameter that reflects the choice of units and is calibrated to the starting price, $A_{V,0} = p_{E,0}/\bar{E}_{-1}^{\psi}$.

5 Quantitative Results

In this section, I investigate the impacts of environmental policy in the quantitative model. In all analyses, the economy is on the laissez-faire BGP (LFBGP) in 2005. Policies or

[28]See table 7.1 in their work. This estimate is in terms of primary energy. Converting primary to final-use energy using the conversion factors described in Appendix Section C.1 yields $\psi = 1.06$ instead of $\psi = 1.26$. Given that these estimates are relatively similar and it is not clear how accurate transformation data are for historical use, I use the latter estimate of ψ and consider robustness over a wide range of alternate values.

[29]I conduct robustness analyses using the extreme values of $\psi = 0$ (exogenous energy prices) and $g_{A_V} = 0$. In the latter case, increases in energy prices are driven entirely by the convexity of the extraction cost curve, which gives $\psi = 3.64$. When comparing the DTC model developed here to the Cobb-Douglas approach from the existing literature, I perform an additional robust check where energy prices are exogenous and constant, implicitly capturing the case where technological progress in energy extraction always endogenously offsets the curvature in the extraction function.

[30]This procedure assumes that ψ is time-invariant.

exogenous shocks begin to affect the economy in 2015.[31]

5.1 Achieving an Exogenous Environmental Target

In this section, I investigate the policies that are necessary to achieve an exogenously given environmental target based on the Paris Agreement.[32] Under the Paris Agreement, the United States aims to reduce carbon emissions to 80 percent below 1990 levels by 2050 (Heal, 2017). In a report for the United Nations, Williams et al. (2014) examine the technical feasibility of achieving this goal. Across a wide range of scenarios, they find that final-use energy consumption must fall over the next several decades. This is true even though almost all electricity is generated from renewable sources by 2050 in their analyses, and the share of final-use energy coming from electricity more than doubles.

While country-level policy targets are often expressed in terms of flows of carbon emissions, climate change is a function of the stock of carbon dioxide in the atmosphere. The broader goal of the Paris Agreement is to keep global average temperature from rising more than 2°C above preindustrial levels.[33] I use the findings of Williams et al. (2014) to construct a target for cumulative energy use between 2015 and 2114. Their work provides estimates of acceptable flows of energy use over the period 2015-2049. Flow energy use declines over this period. To extend the analysis through 2114, I assume that this decline can end abruptly in 2050 and that the growth of energy use can return to LFBGP rates by 2070, all while still being consistent with long-run environmental policy goals.[34] The resulting target is that cumulative energy use over the ten periods from 2015 to 2105 can be 9.3 times the flow energy use in the 2005 period: $E^{\text{target}} = 9.3 \cdot E_{2005}$. Along the LFBGP, cumulative energy use would be over 14 times greater than E_{2005}.

5.1.1 Policy Designed with the Cobb-Douglas Model

In this section, I compare the directed technical change (DTC) model developed in this paper to the Cobb-Douglas approach that is standard in the existing literature. To do so,

[31] When simulating the least-cost path in the DTC and Cobb-Douglas models, I approximate the infinite horizon problem with a finite horizon problem of 1000 years. This has no impact on the outcomes during the period I study. See Appendix Sections B.8.2 and B.9 for details.

[32] As in Section 3.6, I focus on a target for cumulative energy use and abstract from substitution between energy sources. This allows me to focus on the dynamics of energy use, which are the focus of this paper.

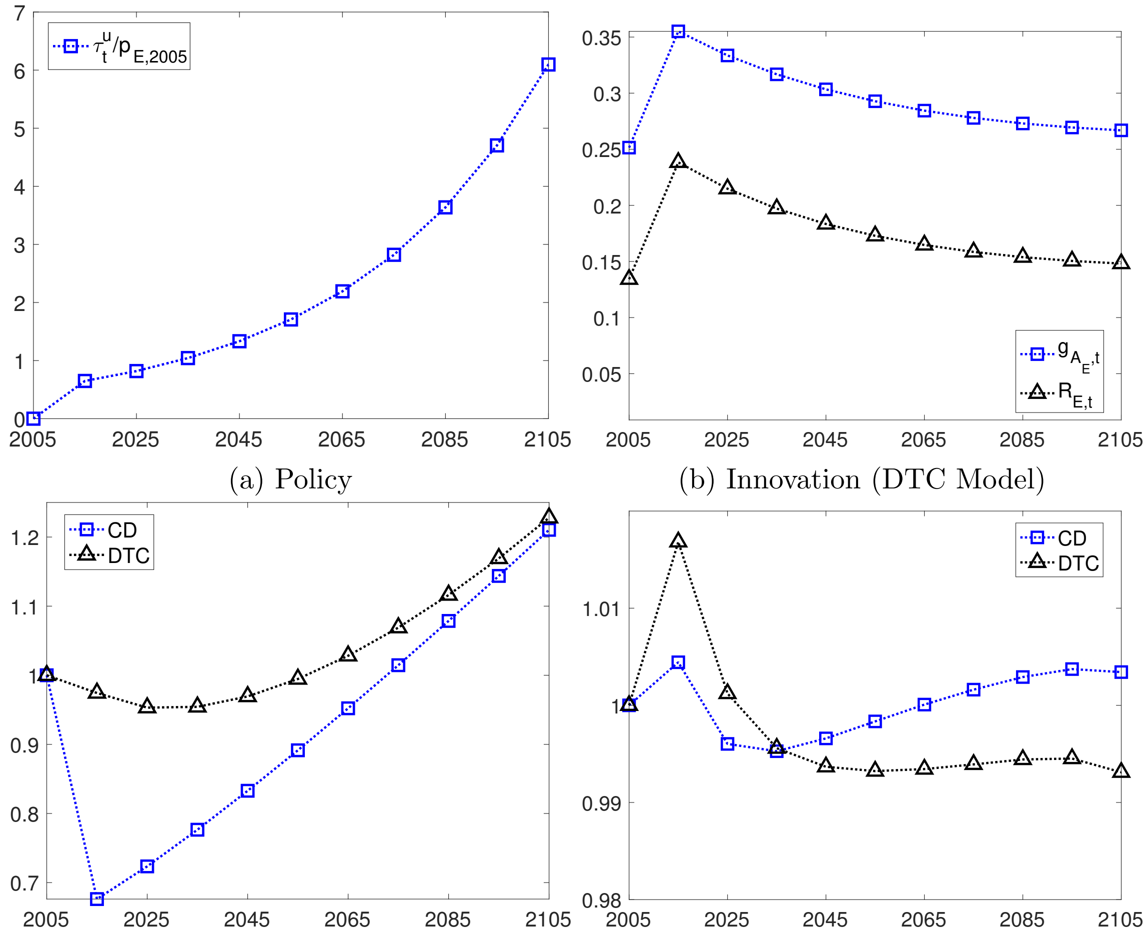
[33] A recent report by the Intergovernmental Panel on Climate Change (IPCC) stresses the importance of reduced final-use energy consumption for keeping global average temperature from rising more than 1.5°C (Rogelj et al., 2018).

[34] See Appendix Section C.2 for further details on the construction of the target.

I build an alternate model that combines the Cobb-Douglas production function with the increasing energy extraction cost specification (4) and the lifetime utility function (11). I calibrate the Cobb-Douglas model so that output, energy use, energy prices, and the energy intensity of output match the DTC model along the LFBGP. The two models also have the same growth rates for all macroeconomic variables along the LFBGP (see Appendix Section B.9 for details). I then find the path of per-unit energy taxes, $\{\tau_t^u\}_{t=0}^{\infty}$, that implements the least-cost path to achieve the energy use target in the Cobb-Douglas model and examine the impact of the same taxes in the DTC model. The thought experiment is: "Suppose policy is designed with models that employ the usual Cobb-Douglas assumption, but reality actually follows the DTC model. How close will the economy come to meeting the policy goals?"

Figure 7 presents the results. Panel (7a) shows the path of per-unit energy taxes that implements the least-cost path in the Cobb-Douglas model. They are normalized by the initial price of energy ($\tau_t^u/p_{E,2005}$). Panel (7b) shows that applying these same energy taxes in the DTC model significantly increases the share of R&D inputs devoted to energy efficiency ($R_{E,t}$) and therefore the growth rate of energy efficiency ($g_{A_E,t}$). Since the tax makes energy more expensive, it increases the demand for energy efficient capital goods, which in turn alters the profit-maximizing allocation of R&D inputs. Panel (7c) shows the path of energy use in both models, relative to the common 2005 level. Energy use adjusts more quickly in the Cobb-Douglas model. Given the slow dynamics of technology, the decline in energy use is smaller and more protracted in the DTC model. After the initial decrease in 2015, energy use in the Cobb-Douglas model increases monotonically and almost catches back up to the DTC model by the end of the century. The total reduction in cumulative energy use is substantially smaller in the DTC model, which misses the target by approximately 12.4 percent (energy use from 2015-2105 is slightly over 10 times greater than the 2005 level). This difference is driven by the transition dynamics following the implementation of the tax, rather than a difference in long-run flow energy use. These results hold even though extraction costs, and therefore tax-inclusive prices, are lower in the Cobb-Douglas model.

Panel (7d) shows consumption in the two models relative to the LFBGP. In both models, consumption initially spikes and then decreases. The initial spike occurs because the energy tax decreases the return to capital in future periods and consequently the incentive to save. By 2105, the Cobb-Douglas model has higher consumption along the least-cost path than on the LFBGP. Holding all else equal, using policy to achieve the environmental target decreases consumption. But, energy taxes also correct the extraction cost externality, which increases consumption. By 2105, the second force dominates in the Cobb-Douglas model.



(a) Policy (b) Innovation (DTC Model)

(c) Energy (relative to 2005) (d) Consumption

Figure 7: Comparison with Cobb-Douglas (Cobb-Douglas) Model. Outcomes in 2005 correspond to the LFBGP. The government begins implementing the least-cost path in the Cobb-Douglas model in 2015. The energy use target is the following constraint: the sum of flow energy use in the 10 periods from 2015-2105 is less than 9.3 times the flow of energy use in 2005. Panel (a) shows the path of per-unit energy taxes that achieve this target in the Cobb-Douglas model, relative to initial price of energy ($\tau_t^u/p_{E,2005}$). Panel (b) shows innovation outcomes in the DTC model. Panel (c) shows flow energy use relative to the 2005 level in both models. Panel (d) shows consumption relative to the LFBGP level in both models.

Since energy use does not respond as quickly in the DTC model, this second force is muted and consumption declines through 2105. In the very long run, after the target is no longer in effect, consumption increases to about 2 percent above LFBGP levels in the Cobb-Douglas model and 1.5 percent above in the DTC model (Appendix Figure D.9 shows the results through 2305).

To gauge the welfare impact of implementing the least-cost path, I calculate the consumption equivalent variation (CEV). This is the percent increase in LFBGP consumption the representative household would need in every period so that it is indifferent between staying on the LFBGP and moving to path studied in Figure 7. The CEV ignores the environmental consequences of energy use. Implementing the least-cost path has a welfare effect that is equivalent to increasing LFBGP consumption by 0.8 percent in the Cobb-Douglas model.[35] The welfare impact in the DTC model is equivalent to increasing LFBGP consumption by 0.3 percent. However, the comparison between the two models is incomplete, because the environmental target is not met in the DTC model, implying that environmental costs are not held equal.

These results demonstrate that policymakers should take the low short-run elasticity of substitution between energy and non-energy inputs into account when designing climate policy. Otherwise, they will overestimate the reduction in cumulative energy use generated by a given path of taxes. According to the quantitative analysis, ignoring slow transition dynamics will also cause policymakers to underestimate both the initial spike and subsequent fall in consumption generated by a given path of energy taxes. Appendix Section D.3 shows that the main findings from this section are robust to alternate parameter values.

5.1.2 Least-Cost Path in the DTC model

In this section, I study the least-cost path that achieves the environmental target in the DTC model, as well as the policy mix that implements the least-cost path. The analysis highlights the different roles played by energy taxes and R&D subsidies/taxes in implementing the least-cost path. The results are presented in Figure 8. Panel (8a) presents the path of per-unit energy taxes normalized by contemporaneous energy prices ($\tau_t^u/p_{E,t}$) and the path of taxes for energy efficiency R&D normalized by the price of R&D inputs ($-\eta_t^U/p_t^R$).[36] Since the stock of R&D inputs is fixed, a tax on energy efficiency R&D affects the direction of R&D, but not the overall quantity. Thus, it is equivalent to a subsidy for non-energy R&D.

As explained in Section 3.6, separate instruments correct separate market failures along the least-cost path. The energy tax forces the final good producer to internalize the explicit and implicit externalities associated with energy use: increases in future extraction costs and

[35] I approximate lifetime utility with utility over 300 years. The solution to the Cobb-Douglas model requires a 'backstop' technology that halts the growth rate of energy prices. This occurs after 550 years. While the backstop has no effect on the dynamics through 300 years, it makes welfare comparisons uninformative beyond the point at which the dynamics are affected.

[36] Appendix Figure D.8 compares the path of per-unit energy taxes (normalized by $p_{E,2005}$) along the least-cost path in the Cobb-Douglas and DTC models. By 2105, the energy taxes are almost five times higher in the DTC model. This is true for two reasons. First, since cumulative energy use responds more slowly to environmental policy in the DTC model, higher taxes are needed to achieve the energy use target. Second, R&D policy in the DTC model favors non-energy technology and, holding all else equal, promotes energy use. The energy taxes in the DTC model must overcome this opposing force.

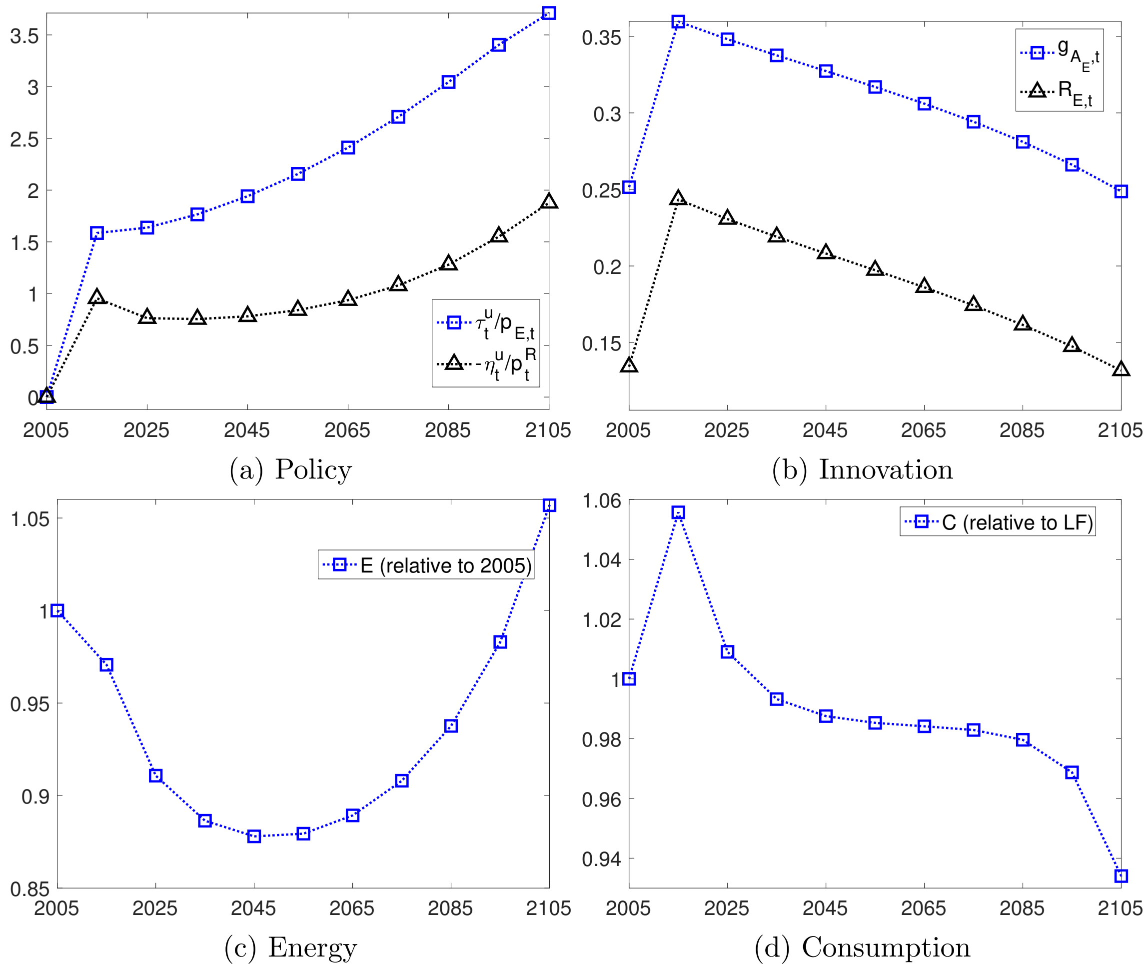


Figure 8: Least-Cost Path. Outcomes in 2005 correspond to the LFBGP. The government begins implementing the least-cost path in 2015. The energy use target is the following constraint: the sum of flow energy use in the 10 periods from 2015-2105 is less than 9.3 times flow energy use in 2005. Panel (a) shows the per-unit energy tax relative to contemporaneous price of energy ($\tau_t^u/p_{E,t}$) and the per-unit tax on energy efficiency R&D relative to the price of R&D inputs ($-\eta_t^U/p_t^R$). Panel (b) shows innovation outcomes, specifically the growth rate of energy efficiency ($g_{A_E,t}$) and the share of R&D inputs dedicated to improving energy efficiency ($R_{E,t}$). Panel (c) shows flow energy use relative to the 2005 level. Panel (d) shows consumption relative to the LFBGP level.

depletion of the energy budget. The energy tax is greater than the extraction cost in 2015 and the ratio increases monotonically while the energy target is in effect. Due to intertemporal knowledge spillovers, the government uses policy in period t to boost R&D in the technology that increases its growth rate between t and $t+1$. Starting in 2015, the first period with environmental policy, this is always non-energy technology, as shown in panel (8b). Relative to the LFBGP, a greater share of R&D resources are allocated to improving energy efficiency, but this outcome does not require subsidies for energy efficiency R&D. Instead, incentives for high energy efficiency R&D come from the energy tax and spillovers from earlier R&D. The fact that implementing the least-cost path requires taxing energy efficiency R&D reinforces the notion that separate instruments correct separate market failures.

Panel (8c) shows the path of energy use. To meet the target, flow energy use between 2015-2095 stays below the 2005 level. Afterwards, energy use rises above the 2005 level, but stays well below the LFBGP level. Compared to the Cobb-Douglas model, the transition dynamics for energy use are slower. Energy use declines until 2045 in the DTC model, whereas the adjustment takes only a single period in the Cobb-Douglas model.

Panel (8d) shows the path of consumption, which spikes at the introduction of policy and subsequently declines while the target is in effect. The path of consumption can be viewed from the perspective of the social planner that chooses the least-cost path or the representative household that reacts to the policy. At all times, $E_t = \frac{K_t^\alpha \left(A_{N,t} L_t\right)^{1-\alpha}}{A_{E,t}}$. So, the social planner can reduce energy use in two ways: (i) decreasing the capital stock by increasing consumption or (ii) redirecting R&D resources towards energy efficiency and away from non-energy technology. Along the least-cost path, they do both. From the perspective of the representative household, the initial spike in consumption is due to the fact that energy taxes decrease the return to capital in future periods and consequently the incentive to save. In the very long run, after the environmental target is no longer binding, consumption reaches about 2 percent above the LFBGP level (Appendix Figure D.10 shows the results through 2305).

Referring back to Figure 7, it is now possible to compare the paths of consumption in the DTC and Cobb-Douglas models when both achieve the environmental target.[37] In the DTC model, the path of consumption again has a higher initial peak and a larger decline by the end of the century. Implementing the least-cost path in the DTC model has an impact on the lifetime utility of the representative household that is equivalent to decreasing LFBGP consumption by 0.3 percent. Implementing the least-cost path has a small positive effect in the Cobb-Douglas model. This implies that the welfare cost of meeting the target is larger when taking into account slow-moving technology dynamics. Appendix Section D.4 shows that the main findings from this section are robust to alternate parameter values.

[37]This comparison ignores the role of clean energy. The welfare cost of reducing energy use is higher in the DTC model. So, meeting a cumulative carbon emissions target in the DTC model would involve more substitution from dirty to clean energy sources, if that margin was included.

5.2 Rebound

5.2.1 Cost-less Technology Shock

In this section, I consider the case where a one-time, unexpected shock causes $A_{E,2015}$ to be 10 percent higher than expected, a 'cost-less technology shock'. The shock occurs after firms make their R&D decisions. The primary goal of the analysis is to examine the impact this exogenous shock has on the subsequent dynamics of R&D and technology, a question that has been overlooked in the existing literature (Gillingham et al., 2016).

Rebound occurs when endogenous reactions either lessen or magnify the partial equilibrium decrease in flow energy use caused by the initial increase in energy efficiency. In a world without rebound, the shock would cause energy efficiency to be 10 percent higher in every period and flow energy use would be $1 - 1/1.1 \approx 9$ percent lower, compared to a world without the original shock.

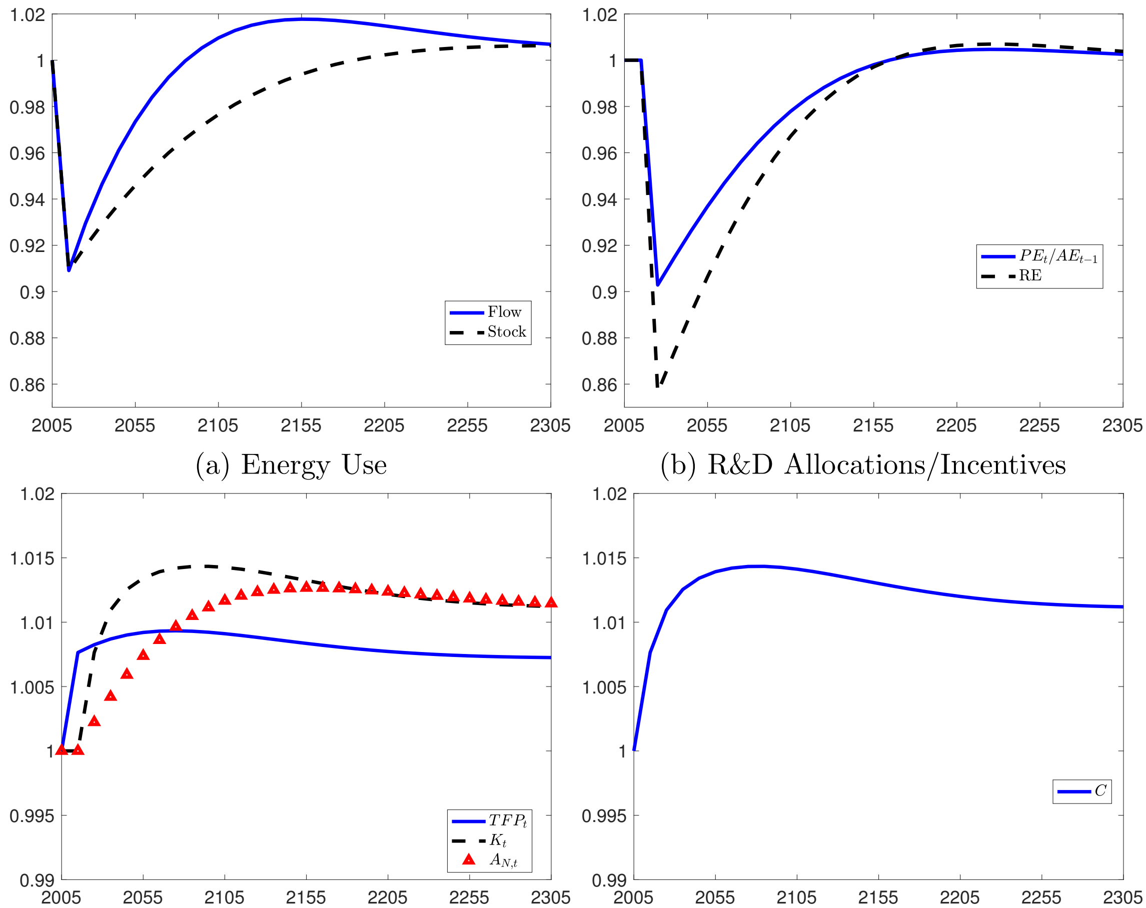
The economy is on a LFBGP prior to the shock. I refer to this as the 'original' LFBGP to distinguish it from the LFBGP that the economy converges to after the shock. The results are shown in Figure 9. All outcomes are shown relative to a baseline case where the economy remains on the original LFBGP.

Panel (9a) shows the dynamics of flow and cumulative energy use. There is no immediate rebound. As discussed in the literature on aggregate static models, this is because of Leontief production (e.g., Saunders, 2008). In the first period after the shock, rebound begins. Eventually, flow and cumulative energy use converge to a slightly (≈ 0.5 percent) higher level than in the absence of the shock, an extreme form of rebound known as 'backfire'. Prior to that, flow energy use overshoots the long-run level with a peak that is almost 2 percent higher than the original LFBGP level. The overshoot is driven by endogenous energy price dynamics.[38] Cumulative energy use monotonically converges to its new long-run level. The transition dynamics are slow. It takes 80 years for flow energy use to reach the new LFBGP level and approximately two centuries for cumulative energy use to reach the new LFBGP level.

Panel (9b) shows the R&D allocation, $R_{E,t}$, as well as $p_{E,t}/A_{E,t-1}$, which governs R&D incentives. The increase in $A_{E,t}$ decreases the energy expenditure share and incentives for energy efficiency R&D in subsequent periods. As a result, $R_{E,t}$ falls below the original LFBGP level in the period after the shock. This reduction in energy efficiency R&D slows

[38] Appendix Section D.5.1 shows the results with exogenous energy prices that grow at the LFBGP level. In this case, overshoot disappears. Appendix Section B.7.2 shows the stability results, which indicate that the economy converges monotonically to the LFBGP with exogenous energy prices, but converges non-monotonically with endogenous energy prices.

the growth of $A_{E,t}$ and increases the growth of $A_{N,t}$, both of which contribute to rising flow energy use. In the long run, the R&D allocations and incentives return to their original LFBGP levels. This is consistent with the theoretical results from Section 3.5, which show that long-run R&D allocations are determined by the growth rate of tax-inclusive energy prices and R&D market clearing, neither of which is permanently affected by the shock.



(a) Energy Use (b) R&D Allocations/Incentives

(c) Growth Determinants (d) Consumption

Figure 9: Cost-less Technology Shock. The figure shows the impact of an unexpected shock that causes $A_{E,2015}$ to be 10% higher than it would be on the original LFBGP. All results are presented relative to a baseline case where the economy remains on the original LFBGP.

Panel (9c) shows the evolution of several variables that are important for economic growth and help explain the dynamics of energy use and consumption. The fall in $p_{E,t}/A_{E,t}$ and the increase in $A_{N,t}$ both contribute to pushing TFP above its original LFBGP level. The increase in TFP leads to a larger capital stock than on the original LFBGP, another force contributing to rebound.

Panel (9d) shows the dynamics of consumption. In the absence of energy taxes, the DTC model has a constant savings rate, as in Golosov et al. (2014). Following the shock, consumption initially jumps above its original LFBGP level along with TFP, and it continues to increase relative to the original LFBGP due to capital accumulation. Consumption increases relative to the original LFBGP in both the short and long run, even in the presence of backfire. This result highlights the fact that rebound is not a measure of welfare, a point stressed by Chan and Gillingham (2015) and overlooked in much of the earlier literature. Since the energy expenditure share is small, the variation in output and consumption induced by the shock is much smaller than the variation in energy use.

These results complement existing applied microeconomic analyses, which focus on rebound over relatively short periods and find results suggesting that backfire is unlikely (Borenstein et al., 2015; Gillingham et al., 2016). The model suggests that these studies accurately capture the rebound in the short run, but understate rebound in the long run, because they ignore the slow-moving process of technical change.[39] More generally, the results demonstrate the importance of studying the entire transition path. Focusing only on the long run ignores the fact that energy efficiency improvements decrease energy use relative to the original LFBGP for decades before backfire occurs.

5.2.2 Permanent Subsidy

I now consider a permanent 40 percent subsidy to energy efficiency R&D, which increases $A_{E,2015}$ by approximately 10 percent, making it easy to compare with the results in the previous section.[40] The analysis serves two purposes. First, it extends the theoretical results in Proposition 2, which says nothing about the impact of a constant subsidy on the level of energy use, the speed of the transition, or the response of consumption. Second, the analysis presented here provides a more policy-oriented examination of rebound when compared to the previous section.

Panel (10a) shows the path of flow and cumulative energy use. The subsidy permanently reduces both flow and cumulative energy use relative the original LFBGP. In other words, backfire does not occur. Indeed, the long-run reduction in energy use is larger than the immediate reduction. The transition dynamics are slow. Flow energy use has not reached

[39]Recent advances in the literature study rebound in general equilibrium models that account for reallocation between sectors, a force that is missing from the DTC model presented here (Fullerton and Ta, 2020; Lemoine, 2020a; Blackburn and Moreno-Cruz, 2021). These models do not take into account how a cost-less technology shock will affect the subsequent dynamics of $A_{N,t}$ and $A_{E,t}$, suggesting that they also underestimate the rebound over long time horizons.

[40]Appendix Section D.5.2 shows the results from a single-period subsidy and explains how they can be understood as a mix of the cost-less technology shock and permanent subsidy results.

its new LFBGP level by 2305. As in the case of the cost-less technology shock, there is some non-monotonicity in flow energy use relative to the original LFBGP, which is driven by the endogenous energy price dynamics.[41]

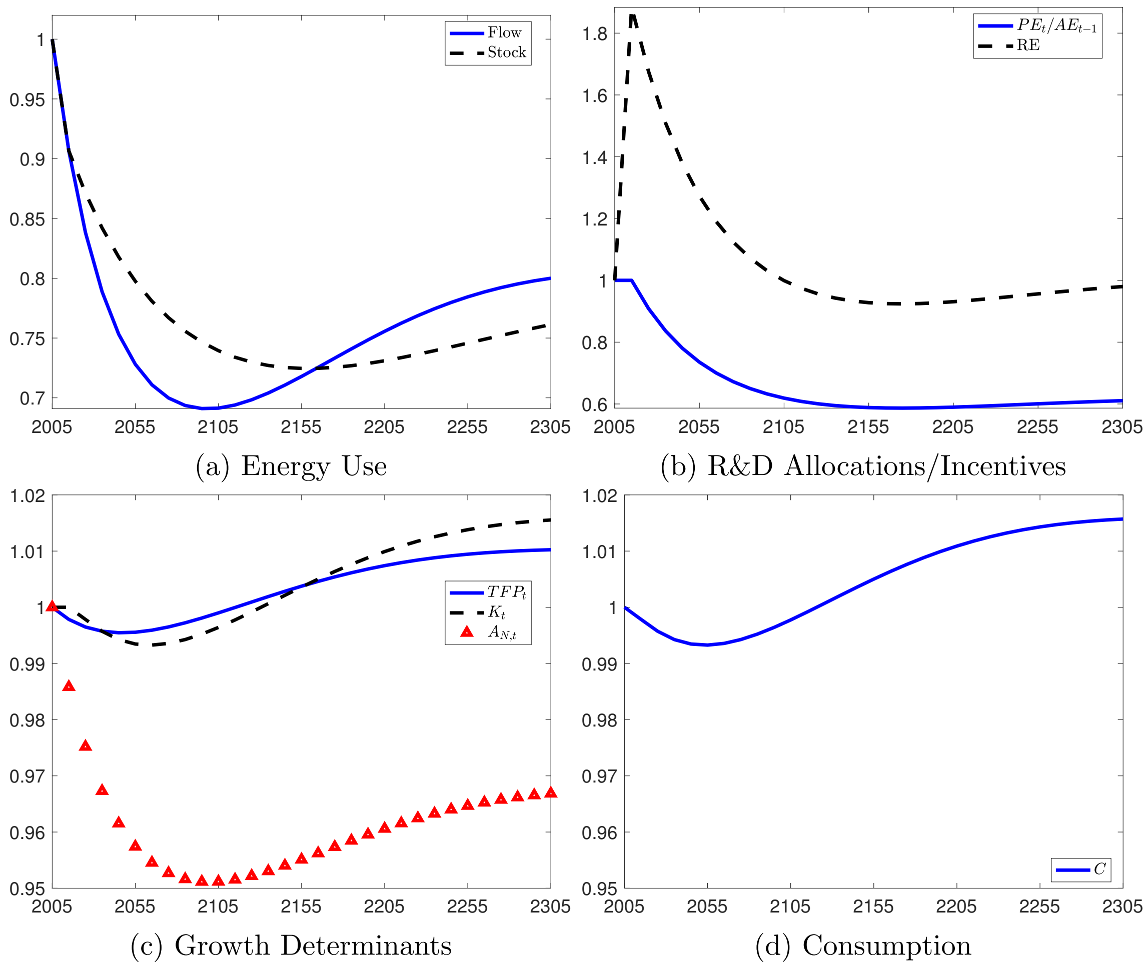


Figure 10: Permanent Subsidy. Impact of a permanent subsidy $\eta_t^S = 40\% \ \forall t$. This causes $A_{E,2015}$ to be 10% higher than the LFBGP level. All results are presented relative to the LFBGP.

Panel (10b) shows the evolution of R&D incentives and allocations. Subsidies increase incentives for energy efficiency R&D, conditional on $p_{E,t}/A_{E,t-1}$. In 2015, therefore, $R_{E,t}$ rises above its original LFBGP level, even as $p_{E,t}/A_{E,t-1}$ is unchanged. Since $A_{E,t}$ grows faster than it would on the original LFBGP, $p_{E,t}/A_{E,t-1}$ falls below the original LFBGP level in the next period. Energy efficiency R&D is lower than in the initial period of the subsidy, but still higher than it would be on the original LFBGP. This process continues until R&D allocations return to their original LFBGP levels. There is a slight overshoot due

[41]Appendix Section D.5.3 presents the results with exogenous energy prices and shows that there is no non-monotonicity.

to the endogenous energy price dynamics.

Panel (10c) presents the dynamics of TFP_t, K_t, and $A_{N,t}$. Unlike the cost-less technology shock case, the subsidy increases $g_{A_E,t}$ at the expense of $g_{A_N,t}$. In the short run, the net effect of the subsidy is to push TFP and capital below their original LFBGP levels. This relative decline continues until 2145, when the reductions in the price of energy begin to boost TFP. Eventually, TFP and capital rise above their original LFBGP levels. Panel (10d) shows that consumption once again follows the path of TFP and capital.[42]

These results have important implications for the study of rebound. Gillingham et al. (2016) and Fullerton and Ta (2020) argue that analyses of cost-less technology shocks are not necessarily informative about rebound following 'policy-induced' increases in energy efficiency. Here, the subsidy permanently alters R&D incentives, shutting down rebound through this channel and leading to different dynamics than those that occur following a cost-less technology shock. The results also add to the findings of Proposition 2, which shows that constant R&D subsidies alone cannot prevent energy use from growing in the long run. The simulation shows that subsidies can still decrease the level of energy use relative to a world without policy, implying that they could be an important part of a portfolio of strategies aimed at addressing climate change. In addition, the results presented here show that a subsidy can increase consumption in the long run by decreasing cumulative energy use and extraction costs, highlighting the fact that climate change may not be the only externality relevant for evaluating energy efficiency policies.

6 Conclusion

Economic analysis of climate change has benefited substantially from the study of growth models (e.g., Nordhaus and Boyer, 2000; Golosov et al., 2014; Barrage, 2020). This paper contributes to this ongoing effort by building a quantitative, directed technical change model that includes final-use energy and can be used to evaluate a wide range of policies. Final-use energy consumption is just one important facet climate policy. Future work can incorporate this representation of final-use energy into integrated assessment models with multiple sources of energy in order to gain a more complete view of the costs and benefits of climate policy. In addition, it would be interesting to study the directed technical change model developed here in a setting where technology can flow between countries as in Hémous

[42]This non-monontonicity is again driven by endogenous energy prices. In the case of exogenous energy prices, there is no negative externality from energy use. The subsidy permanently decreases TFP, capital, and consumption, as shown in Appendix Section D.5.3.

(2016). The opportunity for technology transfer would create another important difference with the Cobb-Douglas model.

A key insight of this paper is the need to differentiate between the short-run elasticity of substitution and the process of directed technical change when studying the impacts of environmental policy. Both forces push production away from taxed inputs, but technology evolves more slowly. In the context of final-use energy consumption, the existing literature assumes that all of the observed long-run reallocation between energy and non-energy inputs occurs via short-run substitution, while the data suggest that essentially all reallocation is due to the slower-moving process of directed technical change. As a result, accounting for the role of technology slows the adjustment speed of the economy following the introduction of a new energy tax.

While this paper focuses on final-use energy consumption, the findings have broader implications for the relationship between technical change and environmental outcomes. In particular, similar factors likely matter when considering substitution between clean and dirty sources of energy. It is notoriously difficult to separately estimate the short-run elasticities of substitution and directed technical change in any production function (Diamond et al., 1978). Recent work in the macroeconomics of climate change frequently relies on clean-dirty substitution elasticity estimates from one of two sources. The first is Papageorgiou et al. (2017), who assume that all technical change is factor-neutral. The second is a meta-study by Stern (2012), who notes that none of the underlying studies account for the fact that the direction of technical change could be endogenous to changes in prices. Thus, neither of the two primary sources separates the short-run elasticity of substitution from directed technical change. As a result, disentangling these two forces through the modeling of directed technical change will likely slow the speed of substitution between clean and dirty sources of energy in leading macroeconomic models of climate change.

Previous to this paper, the environmental macroeconomics literature has argued that accounting for technical change increases the flexibility of the economy and decreases the level of intervention necessary to achieve exogenous policy targets (e.g., Goulder and Schneider, 1999; Popp, 2004; Fried, 2018). This paper reaches the opposite conclusion. The key difference is the treatment of 'off-the-shelf' estimates of the elasticity of substitution between production factors. Previous studies compare models with exogenous and endogenous technical change, holding the short-run elasticity of substitution between inputs fixed. In this case, adding technology as an extra margin of adjustment increases the flexibility of the economy. This is an important observation. But, it is only the relevant metric for updating quantitative outcomes if existing studies with exogenous technology are already using well-identified estimates of the short-run elasticity of substitution. When modeling technology requires disentangling existing elasticity estimates, accounting for the slow-moving process of technical change decreases the flexibility of the economy.

Data Availability Statement

The data and code underlying this research are available on Zenodo at https://doi.org/10.5281/zenodo.7449242.

References

Acemoglu, D. (2002): "Directed technical change," *Review of Economic Studies*, 69, 781–809.

——— (2003): "Labor-and capital-augmenting technical change," *Journal of the European Economic Association*, 1, 1–37.

Acemoglu, D., P. Aghion, L. Bursztyn, and D. Hemous (2012): "The environment and directed technical change," *American Economic Review*, 102, 131–66.

Acemoglu, D., U. Akcigit, H. Alp, N. Bloom, and W. Kerr (2018): "Innovation, reallocation, and growth," *American Economic Review*, 108, 3450–91.

Acemoglu, D., U. Akcigit, D. Hanley, and W. Kerr (2016): "Transition to clean technology," *Journal of Political Economy*, 124, 52–104.

Aghion, P., A. Dechezleprêtre, D. Hémous, R. Martin, and J. Van Reenen (2016): "Carbon taxes, path dependency, and directed technical change: Evidence from the auto industry," *Journal of Political Economy*, 124, 1–51.

Akcigit, U., and W. R. Kerr (2018): "Growth through heterogeneous innovations," *Journal of Political Economy*, 126, 1374–1443.

André, F. J., and S. Smulders (2014): "Fueling growth when oil peaks: Directed technological change and the limits to efficiency," *European Economic Review*, 69, 18–39.

Atkeson, A., and P. J. Kehoe (1999): "Models of energy use: Putty-putty versus putty-clay," *American Economic Review*, 89, 1028–1043.

Barker, T., A. Dagoumas, and J. Rubin (2009): "The macroeconomic rebound effect and the world economy," *Energy Efficiency*, 2, 411.

Barrage, L. (2020): "Optimal dynamic carbon taxes in a climate-economy model with distortionary fiscal policy," *Review of Economic Studies*, 87, 1–39.

Baumeister, C., and L. Kilian (2016): "Forty years of oil price fluctuations: Why the price of oil may still surprise us," *Journal of Economic Perspectives*, 30, 139–160.

Blackburn, C. J., and J. Moreno-Cruz (2021): "Energy efficiency in general equilibrium with input-output linkages," *Journal of Environmental Economics and Management*, 102524.

Bloom, N., C. I. Jones, J. Van Reenen, and M. Webb (2020): "Are ideas getting harder to find?" *American Economic Review*, 110, 1104–44.

Borenstein, S. et al. (2015): "A microeconomic framework for evaluating energy efficiency rebound and some implications," *Energy Journal*, 36, 1–21.

Bosetti, V., C. Carraro, M. Galeotti, E. Massetti, and M. Tavoni (2006): "A world induced technical change hybrid model," *Energy Journal*, 27, 13–37.

Chan, N. W., and K. Gillingham (2015): "The microeconomic theory of the rebound effect and its welfare implications," *Journal of the Association of Environmental and Resource Economists*, 2, 133–159.

Chang, J.-J., W.-N. Wang, and J.-Y. Shieh (2018): "Environmental rebounds/backfires: Macroeconomic implications for the promotion of environmentally-friendly products," *Journal of Environmental Economics and Management*, 88, 35–68.

Covert, T., M. Greenstone, and C. R. Knittel (2016): "Will we ever stop using fossil fuels?" *Journal of Economic Perspectives*, 30, 117–137.

Di Maria, C., and S. Valente (2008): "Hicks meets Hotelling: The direction of technical change in capital–resource economies," *Environment and Development Economics*, 13, 691–717.

Diamond, P., D. McFadden, and M. Rodriguez (1978): "Measurement of the elasticity of factor substitution and bias of technical change," in *Contributions to Economic Analysis* Volume 2: Elsevier, 125–147.

Energy Information Administration (2019a): "Annual Energy Review," https://www.eia.gov/totalenergy/data/annual/ (accessed 8/5/19).

——— (2019b): "State Energy Data System," https://www.eia.gov/state/seds/seds-data-complete.php (accessed 8/5/19).

Fried, S. (2018): "Climate policy and innovation: A quantitative macroeconomic analysis," *American Economic Journal: Macroeconomics*, 10, 90–118.

Fullerton, D., and C. L. Ta (2020): "Costs of energy efficiency mandates can reverse the sign of rebound," *Journal of Public Economics*, 188, 104225.

Gillingham, K., D. Rapson, and G. Wagner (2016): "The rebound effect and energy efficiency policy," *Review of Environmental Economics and Policy*, 10, 68–88.

Gillingham, K., and J. H. Stock (2018): "The cost of reducing greenhouse gas emissions," *Journal of Economic Perspectives*, 32, 53–72.

Golosov, M., J. Hassler, P. Krusell, and A. Tsyvinski (2014): "Optimal taxes on fossil fuel in general equilibrium," *Econometrica*, 82, 41–88.

Goulder, L. H., and S. H. Schneider (1999): "Induced technological change and the attractiveness of CO_2 abatement policies," *Resource and Energy Economics*, 21, 211–253.

Greaker, M., T.-R. Heggedal, and K. E. Rosendahl (2018): "Environmental policy and the direction of technical change," *Scandinavian Journal of Economics*, 120, 1100–1138.

Hart, R. (2013): "Directed technological change and factor shares," *Economics Letters*, 119, 77–80.

——— (2018): "Rebound, directed technological change, and aggregate demand for energy," *Journal of Environmental Economics and Management*, 89, 218–234.

——— (2019): "To everything there is a season: Carbon pricing, research subsidies, and the transition to fossil-free energy," *Journal of the Association of Environmental and Resource Economists*, 6, 349–389.

Hart, R., and D. Spiro (2011): "The elephant in Hotelling's room," *Energy Policy*, 39, 7834–7838.

Hassler, J., P. Krusell, and C. Olovsson (2012): "Energy-saving technical change," *NBER Working Paper No. 18456*.

——— (2021a): "Directed technical change as a response to natural-resource scarcity," *Journal of Political Economy*, 129, 3039–3066.

——— (2021b): "Suboptimal Climate Policy," *Journal of the European Economic Association*.

Hassler, J., P. Krusell, and A. A. Smith Jr (2016): "Environmental macroeconomics," in *Handbook of Macroeconomics* Volume 2: Elsevier, 1893–2008.

Heal, G. (2017): "Reflections: What would it take to reduce US greenhouse gas emissions 80 percent by 2050?" *Review of Environmental Economics and Policy*, 11, 319–335.

Hémous, D. (2016): "The dynamic impact of unilateral environmental policies," *Journal of International Economics*, 103, 80–95.

Hotelling, H. (1931): "The economics of exhaustible resources," *Journal of Political Economy*, 137–175.

International Energy Agency (2018): "IEA Headline Energy Data," https://www.iea.org/data-and-statistics/data-product/world-energy-balances-highlights (accessed 8/5/19).

Jones, C. I. (2016): "The facts of economic growth," in *Handbook of Macroeconomics* Volume 2: Elsevier, 3–69.

Le Quéré, C., J. I. Korsbakken, C. Wilson et al. (2019): "Drivers of declining CO_2 emissions in 18 developed economies," *Nature Climate Change*, 9, 213–217.

Lemoine, D. (2020a): "General equilibrium rebound from energy efficiency innovation," *European Economic Review*, 103431.

——— (2020b): "Innovation-led transitions in energy supply," *Working Paper*.

McGlade, C., and P. Ekins (2015a): "Data for: The geographical distribution of fossil fuels unused when limiting global warming to 2°C," *Nature*, 517, https://doi.org/10.1038/nature14016.

——— (2015b): "The geographical distribution of fossil fuels unused when limiting global warming to 2°C," *Nature*, 517, 187–190.

Metcalf, G. E. (2008): "An empirical analysis of energy intensity and its determinants at the state level," *Energy Journal*, 1–26.

Nagy, B., J. D. Farmer, Q. M. Bui, and J. E. Trancik (2013): "Statistical basis for predicting technological progress," *PloS One*, 8, e52669.

Nemet, G. F. (2006): "Beyond the learning curve: Factors influencing cost reductions in photovoltaics," *Energy Policy*, 34, 3218–3232.

Nordhaus, W. D., and J. Boyer (2000): *Warming the world: Economic models of global warming*: MIT press.

Organization for Economic Cooperation and Development (2019): "Gross domestic spending on RD (indicator)," doi: 10.1787/d8b068b4-en (accessed 8/6/19).

Papageorgiou, C., M. Saam, and P. Schulte (2017): "Substitution between clean and dirty energy inputs: A macroeconomic perspective," *Review of Economics and Statistics*, 99, 281–290.

Peretto, P. F. (1998): "Technological change and population growth," *Journal of Economic Growth*, 3, 283–311.

Performance Curve Database (2019): "Photovoltaics 2," https://pcdb.santafe.edu/index.php (accessed 8/7/19).

Peters, G. P., R. M. Andrew, J. G. Canadell, S. Fuss, R. B. Jackson, J. I. Korsbakken, C. Le Quéré, and N. Nakicenovic (2017): "Key indicators to track current progress and future ambition of the Paris Agreement," *Nature Climate Change*, 7, 118.

Pindyck, R. S. (1978): "The optimal exploration and production of nonrenewable resources," *Journal of Political Economy*, 841–861.

van der Ploeg, F., and C. Withagen (2012): "Too much coal, too little oil," *Journal of Public Economics*, 96, 62–77.

Popp, D. (2002): "Induced innovation and energy prices," *American Economic Review*, 92, 160–180.

——— (2004): "ENTICE: Endogenous technological change in the DICE model of global warming," *Journal of Environmental Economics and Management*, 48, 742–768.

Raupach, M. R., G. Marland, P. Ciais, C. Le Quéré, J. G. Canadell, G. Klepper, and C. B. Field (2007): "Global and regional drivers of accelerating CO_2 emissions," *Proceedings of the National Academy of Sciences*, 104, 10288–10293.

Rogelj, J., D. Shindell, K. Jiang et al. (2018): "Mitigation pathways compatible with 1.5°C in the context of sustainable development," in *Global warming of 1.5°C*: Intergovernmental Panel on Climate Change (IPCC), 93–174.

Rogner, H.-H., L. A. LA, R. F. Aguilera et al. (2012): "Energy Resources and Potentials," in *Global energy assessment: Toward a sustainable future* ed. by Johansson, T. B., Patwardhan, A. P., Nakićenović, N., and Gomez-Echeverri, L.: Cambridge University Press.

Saunders, H. D. (1992): "The Khazzoom-Brookes postulate and neoclassical growth," *Energy Journal*, 131–148.

——— (2000): "A view from the macro side: Rebound, backfire, and Khazzoom–Brookes," *Energy Policy*, 28, 439–449.

——— (2008): "Fuel conserving (and using) production functions," *Energy Economics*, 30, 2184–2235.

Slade, M. E. (1982): "Trends in natural-resource commodity prices: An analysis of the time domain," *Journal of Environmental Economics and Management*, 9, 122–137.

Smulders, S., and M. De Nooij (2003): "The impact of energy conservation on technology and economic growth," *Resource and Energy Economics*, 25, 59–79.

Stern, D. I. (2012): "Interfuel substitution: A meta-analysis," *Journal of Economic Surveys*, 26, 307–331.

Sue Wing, I. (2008): "Explaining the declining energy intensity of the US economy," *Resource and Energy Economics*, 30, 21–49.

Turner, K. (2009): "Negative rebound and disinvestment effects in response to an improvement in energy efficiency in the UK economy," *Energy Economics*, 31, 648–666.

Uzawa, H. (1961): "Neutral inventions and the stability of growth equilibrium," *Review of Economic Studies*, 28, 117–124.

Van der Werf, E. (2008): "Production functions for climate policy modeling: An empirical analysis," *Energy Economics*, 30, 2964–2979.

Williams, J. H., B. Haley, F. Kahrl, J. Moore, A. D. Jones, M. S. Torn, H. McJeon et al. (2014): "Pathways to deep decarbonization in the United States," *The US Report of the Deep Decarbonization Pathways Project of the Sustainable Development Solutions Network and the Institute for Sustainable Development and International Relations, Energy and Environmental Economics, San Francisco, CA, accessed Apr*, 23, 2016.

Online Appendix

Energy Efficiency and Directed Technical Change: Implications for Climate Change Mitigation

By: Gregory Casey

A Data Sources and Definitions

A.1 Figure 1

Primary Energy (E_p). Total energy extracted from the environment (i.e., production) plus net imports. For renewables used in electricity generation, production is equal to electricity generated. Measured in kilotonnes of oil equivalent (ktoe). Data cover 1971-2016. Source: 'IEA Headline Energy Data'. Accessed 8/5/19 from http://www.iea.org/statistics/topics/energybalances/.[43]

Final-Use Energy (E_f). Total energy consumption: total primary energy minus losses occurring during transformation and energy industry own use. Measured in ktoe. Data cover 1971-2016. Accessed 8/5/19. Source: see *Primary Energy*.

Carbon Dioxide Emissions (CO_2). Carbon dioxide emissions from fuel combustion. Measured in megatonnes (Mt). Data cover 1971-2016. Accessed 8/5/19. Source: see *Primary Energy*.

Real GDP (Y). Real gross domestic product in 2012 chained dollars. Data cover 1949-2018. Source: NIPA. Accessed 8/5/19 via 'Table C1: Population, U.S. gross domestic product, and U.S. Gross Output' at https://www.eia.gov/totalenergy/data/annual/.

Price of Solar Energy. Real levelized cost of electricity (\$2005/kWh) produced from photovoltaic (PV) modules in the United States. Data cover 1977-2009. Original source: Nemet (2006). Accessed 8/5/19 via the Performance Curve Database from the Sante Fe Institute (Nagy et al., 2013), which includes updated data through 2009. See 'Photovoltaics 2' at

[43]The IEA data have moved and are now available as 'World Energy Balances Highlights' at https://www.iea.org/data-and-statistics/data-products. Access now requires registration.

http://pcdb.santafe.edu/index.php.

Nominal Energy Price. Nominal average price of energy paid by end users in the United States. Measured in dollars per million BTU. Data cover 1970-2017. Due to data limitations, prices for energy derived from renewable sources are not included.[44] Accessed 8/5/19. Source: 'Total energy prices and expenditures' at https://www.eia.gov/state/seds/seds-data-complete.php.

GDP Deflator. GDP implicit price deflator with base year 2012. Data cover from 1949-2018. Accessed 8/5/19. Source: see *Real GDP*.

Real Energy Price. Average real price of primary energy in 2012 chained dollars. Author's calculation: *Nominal Energy Price* divided by *GDP Deflator*.

A.2 Figure 2

See **Real GDP**, **Nominal Energy Price**, and **Real Energy Price** from Figure 1.

Nominal Energy Expenditure. Nominal energy expenditure in the United States. Due to price data limitations, spending on final-use energy derived from renewable sources is not included. Accessed 8/5/19. Source: see *Nominal Energy Price*.

Nominal GDP (Y). Nominal gross domestic product. Data available from 1929-2018. Accessed 8/5/19. Source: see *Real GDP*.

Energy Expenditure Share (E_{share}). Author's calculation: *Nominal Energy Expenditure* divided by *Nominal GDP*.

Energy Use. Final-use energy consumption. Author's calculation: *Nominal Energy Expenditure* divided by *Nominal Energy Price*.

Energy Intensity of Output (E/Y). Total final-use energy consumption per real dollar of GDP. Author's calculation: *Energy Use* divided by *Real GDP*.

[44] Documentation is available at https://www.eia.gov/state/seds/seds-technical-notes-complete.php. See Section 7 of 'Prices and expenditures'.

A.3 Figure 3

See **Primary Energy** and **Final-Use Energy** from Figure 1.

Oil. Total primary energy from oil, natural gas liquids, and feedstocks. Measured in kilotonnes of oil equivalent (ktoe). Data cover 1971-2016. Accessed 8/5/19. Source: see *Primary Energy*.

Renewable Energy. Total primary energy from renewable sources and waste. Data cover from 1971-2016. Accessed 8/5/19. Source: see *Primary Energy*.

Non-renewable Energy. Total primary energy from non-renewable sources. Author's calculation: *Primary Energy* minus *Renewable Energy*.

Energy Extraction Costs. Estimates of available fossil fuel energy resources remaining in the environment, and the cost of extracting those resources. Costs and availability are measured in terms of final-use energy that can eventually used from primary resources. The original estimates come from McGlade and Ekins (2015b), who focus on primary energy availability and corresponding extraction costs. I use conversion factors from Rogner et al. (2012) to convert heterogeneous primary energy sources into common units, and data from the IEA and EIA to estimate efficiency of transforming primary energy into final-use energy. Appendix Section C.1 provides further detail on the calculations. Accessed 8/7/19. Further background is available in McGlade (2014). Data available from: `https://www.nature.com/articles/nature14016` (source data for table 1).

A.4 Calibration

See above for details regarding **Real GDP**, **Energy Use**, **Energy Expenditure Share**, and **Energy Extraction Costs**.

Population. Total resident population of the United States. Data cover 1949-2018. Accessed 8/5/19. Source: see *Real GDP*.

R&D Share. Share of GDP devoted to research and development. Data cover 1981-2017. Source: Bureau of Economic Analysis. Accessed 8/6/19 via the OECD:

https://data.oecd.org/rd/gross-domestic-spending-on-r-d.htm.

B Derivations of Model Equations and Results

B.1 Final Good Producer

The final good producer uses the Leontief production function (2). It chooses labor, capital, and energy inputs to maximize profits subject to constraint (3) for each i. The price of the final good is normalized to one. Let $\upsilon_t(i)$ be the Lagrange multiplier attached to the constraint for capital good i. The resulting Lagrangian is given by:

$$\mathcal{L} = \int_0^1 A_{E,t}(i)E_t(i)di - w_t L_t - \int_0^1 p_{X,t}(i)X_t(i)di - \tau_t p_{E,t}\int_0^1 E_t(i)di - \int_0^1 \upsilon_t(i)\left[A_{E,t}(i)E_t(i) - X_t(i)^\alpha \left(A_{N,t}(i)L_t\right)^{1-\alpha}\right]di. \quad \text{(B.1)}$$

Complementary slackness implies

$$\upsilon_t(i)\left[A_{E,t}(i)E_t(i) - \left(A_{N,t}(i)X_t(i)\right)^\alpha L_t^{1-\alpha}\right] = 0 \;\forall i. \quad \text{(B.2)}$$

The constraints will always bind, because the final good producer would never hire labor, rent capital goods, or purchase energy that went unused. The first order conditions with respect to $E_t(i)$, $X_t(i)$, and L_t are given by:

$$\upsilon_t(i) = 1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}, \quad \text{(B.3)}$$

$$\upsilon_t(i) = \frac{p_{X,t}(i)}{\left(\alpha A_{N,t}L_t\right)^{1-\alpha} X_t(i)^{\alpha-1}}, \quad \text{(B.4)}$$

$$w_t = (1-\alpha)L_t^{-\alpha}\int_0^1 \upsilon_t(i)A_{N,t}(i)^{1-\alpha}X_t(i)^\alpha \, di. \quad \text{(B.5)}$$

Substituting (B.3) into (B.4) and (B.5), respectively, yields the following inverse demand functions:

$$p_{X,t}(i) = \alpha\left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]\left(A_{N,t}(i)L_t\right)^{1-\alpha} X_t(i)^{\alpha-1}, \quad \text{(B.6)}$$

$$w_t = (1-\alpha)L_t^{-\alpha}\int_0^1 \left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right] A_{N,t}(i)^{1-\alpha}X_t(i)^\alpha \, di. \quad \text{(B.7)}$$

B.2 Capital Good Producers

Each capital good producer i chooses production quantities, $X_t(i)$, research inputs, $R_{E,t}(i)$ and $R_{N,t}(i)$, and technology levels, $A_{E,t}(i)$ and $A_{N,t}(i)$, to maximize single period profits subject to inverse demand and research productivity constraints:

$$\max\ \pi_{X,t}(i) = p_{X,t}(i)X_t(i) - (1-\tau_t^K)r_t X_t(i) - (1-\eta_t^S)p_{E,t}^R R_{E,t}(i) - p_{N,t}^R R_{N,t}(i) \quad \text{(B.8)}$$

subject to

$$\begin{aligned} p_{X,t}(i) &= \alpha\left[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]\left(A_{N,t}(i)L_t\right)^{1-\alpha} X_t(i)^{\alpha-1}, & \text{(B.9)} \\ A_{J,t}(i) &= A_{J,t-1}(i) + \eta_J R_{J,t}(i)^{1-\lambda} A_{J,t-1}, \quad J \in \{N,E\}. & \text{(B.10)} \end{aligned}$$

Substitute (B.9) into (B.8) and take the first order condition with respect to $X_t(i)$. Constraint (B.10) is independent of the production level, $X_t(i)$. So, the model yields the standard first order conditions and results, adjusted for the effective cost of energy. In particular,

$$(1-\tau_t^K)r_t = \alpha^2 A_{N,t}(i)^\alpha \left[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]\left(A_{N,t}(i)L_t\right)^{1-\alpha} X_t(i)^{\alpha-1}. \quad \text{(B.11)}$$

Applying $\tau_t^K = (1-\alpha)$, which undoes the monopoly distortion,

$$r_t = \alpha A_{N,t}(i)^\alpha \left[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]\left(A_{N,t}(i)L_t\right)^{1-\alpha} X_t(i)^{\alpha-1}. \quad \text{(B.12)}$$

Rearranging gives

$$X_t(i) = \alpha^{\frac{1}{1-\alpha}} r^{\frac{-1}{1-\alpha}} A_{N,t}(i)L_t \left[1-\frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]^{\frac{1}{1-\alpha}}. \quad \text{(B.13)}$$

Plugging in to (B.9) gives

$$p_{X,t}(i) = r_t. \quad \text{(B.14)}$$

With (B.13) and (B.14), it is possible to rewrite the the capital good producer problem as one of choosing research inputs and technology levels to maximize profits:

$$\max \pi_{X,t}(i) = \tilde{\alpha} r_t^{\frac{-\alpha}{1-\alpha}} A_{N,t}(i) L_t \left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]^{\frac{1}{1-\alpha}} - (1-\eta_t^S) p_{E,t}^R R_{E,t}(i) - p_{N,t}^R R_{N,t}(i) \quad \text{(B.15)}$$

subject to

$$A_{J,t}(i) = A_{J,t-1}(i) + \eta_J R_{J,t}(i)^{1-\lambda} A_{J,t-1}, \quad J \in \{N, E\}, \quad \text{(B.16)}$$

where $\tilde{\alpha} = (1-\alpha)\alpha^{\frac{1}{1-\alpha}}$. Let κ_J be the Lagrange multiplier for constraint (B.16). The first order conditions for R&D inputs and technology levels yield

$$\begin{aligned}
p_{N,t}^R &= \kappa_N (1-\lambda) \eta_N R_{N,t}(i)^{-\lambda} A_{N,t-1}, && \text{(B.17)}\\
(1-\eta_t^S) p_{E,t}^R &= \kappa_E (1-\lambda) \eta_E R_{E,t}(i)^{-\lambda} A_{E,t-1}, && \text{(B.18)}\\
\kappa_N &= \tilde{\alpha} r_t^{\frac{-\alpha}{1-\alpha}} L_t \left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]^{\frac{1}{1-\alpha}}, && \text{(B.19)}\\
\kappa_E &= \alpha^{\frac{1}{1-\alpha}} r_t^{\frac{-\alpha}{1-\alpha}} A_{N,t}(i) L_t \left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]^{\frac{1}{1-\alpha}-1} \tau_t p_{E,t} A_{E,t}^{-2}. && \text{(B.20)}
\end{aligned}$$

Putting these together,

$$\begin{aligned}
p_{N,t}^R &= \iota_t (1-\alpha) \left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]^{\frac{1}{1-\alpha}} \eta_N R_{N,t}(i)^{-\lambda} A_{N,t-1}, && \text{(B.21)}\\
(1-\eta_t^S) p_{E,t}^R &= \iota_t A_{N,t}(i) \tau_t p_{E,t} A_{E,t}(i)^{-2} \left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}(i)}\right]^{\frac{1}{1-\alpha}-1} \eta_E R_{E,t}(i)^{-\lambda} A_{E,t-1}, && \text{(B.22)}
\end{aligned}$$

where $\iota_t \equiv \alpha^{\frac{1}{1-\alpha}}(1-\lambda) r_t^{\frac{-\alpha}{1-\alpha}} L_t$ is common to both terms. Taking the ratio of (B.22) and (B.21) yields (17) in the main text.

B.3 Aggregation across Capital Good Producers

Plugging constraint (B.16) into (B.13), (B.21), and (B.22) eliminates the contemporary technology terms, leaving production quantities and R&D input allocations as the only endogenous variables chosen by capital good producers at time t. Other than the three endogenous variables, $X_t(i)$, $R_{E,t}(i)$, and $R_{N,t}(i)$, the only terms in these equations that could differ across i are $A_{J,t-1}(i)$ for $J = N, E$. Since it is assumed that initial technology levels are the same prior to period-0 R&D – i.e., $A_{J,-1}(i) = A_{J,-1}\ \forall i$ – all of the equations are symmetric across i, and capital good producers make identical decisions in every period. Since there is a unit mass of producers, this implies that $X_t(i) = X_t$, $R_{J,t}(i) = R_{J,t} \equiv \int_0^1 R_{J,t}(i)di$ and $A_{J,t}(i) = A_{J,t} \equiv \int_0^1 A_{J,t}(i)di \; \forall i, J, t$. Given that the Leontief constraint (3) always holds with equality, this also implies that the final good producer will choose $E_t(i) = E_t \equiv \int_0^1 E_t(i)di$ $\forall i, t$. Finally, full depreciation ensures that capital market clearing (10) holds with equality, and $X_t(i) = K_t = \int_0^1 X_t(i) \; \forall i, t$. With this symmetry, integrating (2) across i gives

$$Q_t = A_{E,t}E_t = K_t^{\alpha} \left(A_{N,t}L_t\right)^{1-\alpha}, \tag{B.23}$$

and similarly from (7), aggregate final output is

$$Y_t = \left[1 - \frac{p_{E,t}}{A_{E,t}}\right] K_t^{\alpha} \left(A_{N,t}L_t\right)^{1-\alpha}. \tag{B.24}$$

B.4 Factor Prices and Factor Shares

In this section, I derive factor prices, profits, and factor shares at the aggregate level. Integrating (B.7) over i gives

$$w_t = (1-\alpha)\left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}}\right] K_t^{\alpha} A_{N,t}^{1-\alpha} L_t^{-\alpha} \tag{B.25}$$

and, therefore,

$$w_t L_t = (1-\alpha)\left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}}\right] K_t^{\alpha} \left(A_{N,t}L_t\right)^{1-\alpha} = \tilde{\tau}_t (1-\alpha) Y_t, \tag{B.26}$$

where $\tilde{\tau}_t = \frac{[1-\frac{\tau_t p_{E,t}}{A_{E,t}}]}{[1-\frac{p_{E,t}}{A_{E,t}}]}$ is the wedge in factor prices caused by energy taxes. Without energy taxes, $\tilde{\tau}_t = 1$. Next, from (B.13) and (B.14),

$$r_t = \alpha\left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}}\right] K_t^{\alpha-1} \left(A_{N,t}L_t\right)^{1-\alpha}, \tag{B.27}$$

$$r_t K_t = \alpha\left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}}\right] K_t^{\alpha} \left(A_{N,t}L_t\right)^{1-\alpha} = \tilde{\tau}_t \alpha Y_t. \tag{B.28}$$

Combining (B.25) and (B.27) yields

$$\frac{w_t^{1-\alpha} r_t^{\alpha}}{\hat{\alpha} A_{N,t}^{1-\alpha}} = \left[1 - \frac{\tau_t p_{E,t}}{A_{E,t}}\right], \tag{B.29}$$

where $\hat{\alpha} = (1-\alpha)^{1-\alpha}\alpha^{\alpha}$. The term on the left is the cost of the capital-labor composite needed to produce on unit of the final good, and $\frac{\tau_t p_{E,t}}{A_{E,t}}$ is the tax-inclusive cost of energy needed to produce one unit of final output. Plugging this result into (17) and applying the aggregation results from Appendix Section B.3 yields (18).

The final good producer also pays taxes on energy expenditure. The tax bill is equal to

$$\begin{aligned}
(\tau_t - 1)p_{E,t}E_t &= (\tau_t - 1)p_{E,t}\frac{Q_t}{A_{E,t}} && \text{(B.30)}\\
&= (\tau_t - 1)\left(\frac{\frac{p_{E,t}}{A_{E,t}}}{1-\frac{p_{E,t}}{A_{E,t}}}\right)Y_t && \text{(B.31)}\\
&= (1-\tilde{\tau}_t)Y_t. && \text{(B.32)}
\end{aligned}$$

The sum of the labor, capital, and energy tax shares of income are equal to one. This occurs because the capital good subsidy undoes the monopoly distortion in the input market. There are still two sources of income, profits of capital good producers and rental rates for R&D inputs. These are paid from the two subsidies received by the capital good producer.

From (B.21) and (B.13), total payments to R&D inputs are given by

$$\begin{aligned}
p_t^R &= (1-\alpha)(1-\lambda)\frac{r_t X_t}{A_{N,t}}\eta_N R_{N,t}^{-\lambda}A_{N,t-1} && \text{(B.33)}\\
&= (1-\lambda)\frac{\eta_N (R_{N,t})^{-\lambda}}{1+g_{N,t}}\cdot\tilde{\tau}_t\alpha(1-\alpha)Y_t. && \text{(B.34)}
\end{aligned}$$

Using (B.13), (B.14), and (B.28), capital good producer profits can be written as

$$\pi_{X,t} = (1-\alpha)r_t K_t + p_t^R \eta_t^S R_{E,t} - p_t^R. \tag{B.35}$$

The first term is total payments to the capital good producer from the subsidy used to undo the monopoly distortion, the second term is total payments from the R&D subsidy, and the third term is total income paid to R&D inputs.

Finally, I turn to discussing the energy expenditure share. In the model, there is no value-added in the energy extraction industry, and final output is defined as gross output minus energy expenditure. Energy expenditure is not a source of income for the representative household. I define the energy expenditure share as total tax-inclusive energy expenditure divided by GDP (see Definition 1). This corresponds to the data shown in Figure 2, where the energy share is also calculated as aggregate expenditure on final-use energy divided by

GDP. Aggregate energy expenditure is:

$$\tau_t p_{E,t} E_t = \tau_t p_{E,t} \frac{Q_t}{A_{E,t}} = \tau_t \left(\frac{\frac{p_{E,t}}{A_{E,t}}}{1 - \frac{p_{E,t}}{A_{E,t}}} \right) Y_t. \tag{B.36}$$

B.5 Aggregate R&D Allocation

B.5.1 Analytic Expression

In this section, I derive an analytic expression for the aggregate R&D allocations. Since R&D inputs are freely mobile across firms and technologies, $p^R_{E,t} = p^R_{N,t} = p^R_t \; \forall t$. Noting the symmetry between capital good producers, (17) can now be rewritten as

$$(1 - \eta^S_t) \frac{A_{E,t}}{A_{E,t-1}} \left[\frac{A_{E,t}}{\tau_t p_{E,t}} - 1 \right] = \frac{A_{N,t}}{A_{N,t-1}} \frac{\eta_E R^{-\lambda}_{E,t}}{(1-\alpha)\eta_N R^{-\lambda}_{N,t}}.$$

Replacing growth rates and technology levels with the values implied by (8) and applying resource constraint (9) yields

$$(1-\eta^S_t)(1+\eta_E R^{1-\lambda}_{E,t}) \left[\frac{(1+\eta_E R^{1-\lambda}_{E,t}) A_{E,t-1}}{\tau_t p_{E,t}} - 1 \right] = (1+\eta_N(1-R_{E,t})^{1-\lambda}) \frac{\eta_E R^{-\lambda}_{E,t}}{(1-\alpha)\eta_N (1-R_{E,t})^{-\lambda}}.$$

Dividing by $(1 - \eta^S_t)$, multiplying through on the left-hand side, and isolating the term with energy prices yields

$$(1+\eta_E R^{1-\lambda}_{E,t})^2 \frac{A_{E,t-1}}{\tau_t p_{E,t}} = \frac{1}{1-\eta^S_t} \left[\frac{\eta_E R^{-\lambda}_{E,t}}{(1-\alpha)\eta_N(1-R_{E,t})^{-\lambda}} \left(1+\eta_N(1-R_{E,t})^{1-\lambda}\right) \right] + (1+\eta_E R^{1-\lambda}_{E,t}).$$

Distributing terms on the right-hand side gives

$$(1+\eta_E R^{1-\lambda}_{E})^2 \frac{A_{E,t-1}}{\tau_t p_{E,t}} = \frac{1}{(1-\alpha)(1-\eta^S_t)} \left[\frac{\eta_E R^{-\lambda}_{E}}{\eta_N(1-R_{E})^{-\lambda}} + \eta_E R^{-\lambda}_E - \eta_E R^{1-\lambda}_{E,t} \right] + (1+\eta_E R^{1-\lambda}_{E}).$$

Now, multiplying through by $\frac{\tau_t p_{E,t}}{A_{E,t-1}}$, taking the square root of both sides, subtracting one, and dividing by $\eta_E R_{E,t}^{-\lambda}$ yields

$$R_{E,t} = \frac{\sqrt{\frac{\tau_t p_{E,t}}{A_{E,t-1}}}\sqrt{\frac{1}{(1-\alpha)(1-\eta_t^S)}\left[\frac{\eta_E R_{E,t}^{-\lambda}}{\eta_N(1-R_{E,t})^{-\lambda}} + \eta_E R_{E,t}^{-\lambda} - \eta_E R_{E,t}^{1-\lambda}\right] + (1+\eta_E R_{E,t}^{1-\lambda})} - 1}{\eta_E R_{E,t}^{-\lambda}}. \tag{B.37}$$

Applying the R&D market clearing condition (9) yields

$$R_{N,t} = 1 - R_{E,t}. \tag{B.38}$$

B.5.2 Implicit Function

In this section, I will show that (B.37) implicitly defines $R_{E,t} = \Gamma(\frac{\tau_t p_{E,t}}{A_{E,t-1}})$ for some function $\Gamma(\cdot)$ with $\Gamma'(\cdot) > 0$. To start, it is helpful to note that (B.37) can be rewritten as

$$1 + \eta_E R_{E,t}^{1-\lambda} = \left(\frac{\tau_t p_{E,t}}{A_{E,t-1}}\right)^{\frac{1}{2}} \Phi(R_{E,t})^{\frac{1}{2}}, \tag{B.39}$$

where $\Phi(R_{E,t}) \equiv \left(\frac{1}{(1-\alpha)(1-\eta_t^S)}\left[\frac{\eta_E R_{E,t}^{-\lambda}}{\eta_N(1-R_{E,t})^{-\lambda}} + \eta_E R_{E,t}^{-\lambda} - \eta_E R_{E,t}^{1-\lambda}\right] + (1+\eta_E R_{E,t}^{1-\lambda})\right)$. Both sides of the equation are continuous. Also, since $R_{E,t} \in [0,1]$, $\Phi(R_{E,t}) > 0$.[45]

Now, I examine each side of the equation separately. Since $\lambda \in [0,1)$, the left-hand side (LHS) of (B.39) is an increasing function of $R_{E,t}$. To examine the right-hand side (RHS) it is helpful to write

$$\begin{aligned}\Phi(R_{E,t}) = &\left(\frac{1}{(1-\alpha)(1-\eta_t^S)}\right)\frac{\eta_E R_{E,t}^{-\lambda}}{\eta_N(1-R_{E,t})^{-\lambda}} \\ &+ \left(\frac{1}{(1-\alpha)(1-\eta_t^S)}\right)\eta_E R_{E,t}^{-\lambda} \\ &+ \left(1 - \frac{1}{(1-\alpha)(1-\eta_t^S)}\right)\eta_E R_{E,t}^{1-\lambda} \\ &+ 1.\end{aligned} \tag{B.40}$$

Note that $\left(1 - \frac{1}{(1-\alpha)(1-\eta_t^S)}\right) < 0$. Line by line inspection of (B.40) reveals that $\Phi'(R_{E,t}) < 0$. We now know that the LHS of (B.39) is increasing in $R_{E,t}$ and the RHS is decreasing in $R_{E,t}$.

[45] To see this, note that $\alpha \in (0,1)$, $\eta_t^S \in [0,1)$, and $\eta_E R_{E,t}^{-\lambda} \geq \eta_E R_{E,t}^{1-\lambda} = \eta_E R_{E,t}^{-\lambda} R_{E,t}$.

So, there is at most one $R_{E,t}$ that solves the equation for any given $\frac{\tau_t p_{E,t}}{A_{E,t-1}}$.

To show that such an $R_{E,t}$ always exists, consider the limits of each side of (B.39) over the set of feasible values for $R_{E,t}$. As $R_{E,t} \to 0$, the LHS $\to 1$. Also, $\lim_{R_{E,t}\to 0} \Phi(R_{E,t}) = \infty$. So, as $R_{E,t} \to 0$, the RHS $\to \infty$. It is also immediate that as $R_{E,t} \to 1$, LHS $\to 1+\eta_E$. Also, $\lim_{R_{E,t}\to 1} \Phi(R_{E,t}) = 1+\eta_E$. So, as $R_{E,t} \to 1$, the RHS $\to \left(\frac{\tau_t p_{E,t}}{A_{E,t-1}}\right)^{\frac{1}{2}} (1+\eta_E)^{\frac{1}{2}} < 1+\eta_E$.[46]

The results are summarized in Figure B.1 below. As $R_{E,t} \to 0$, the RHS is higher than the LHS. But, as $R_{E,t} \to 1$, the LHS is greater than the RHS. Since both sides of the equation are monotonic and continuous, there must be a single $R_{E,t}$ that solves the equation for any $\frac{\tau_t p_{E,t}}{A_{E,t-1}}$. In other words, there exists some well-defined function $R_{E,t} = \Gamma(\frac{\tau_t p_{E,t}}{A_{E,t-1}})$.

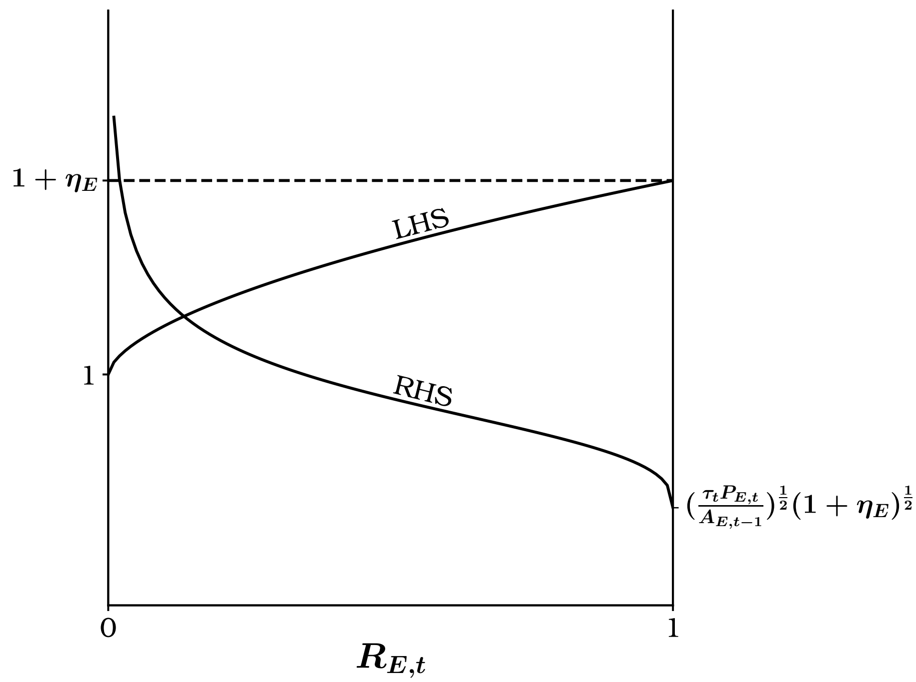


Figure B.1: Determination of $R_{E,t}$. The figure shows both sides of (B.39).

The RHS of (B.39) is increasing in $\frac{\tau_t p_{E,t}}{A_{E,t-1}}$ and decreasing in $R_{E,t}$. The LHS is independent of $\frac{\tau_t p_{E,t}}{A_{E,t-1}}$ and increasing in $R_{E,t}$. Following an increase in $\frac{\tau_t p_{E,t}}{A_{E,t-1}}$, therefore, $R_{E,t}$ must also increase to maintain equality. In other words, $\Gamma'(\cdot) > 0$.

[46] The final inequality is not obvious. First, note that $\frac{\tau p_{E,t}}{A_{E,t}}$ is the share of gross output paid to energy producers. So, in equilibrium, $\frac{\tau p_{E,t}}{A_{E,t}} \in (0,1)$. Also, $\frac{\tau_t p_{E,t}}{A_{E,t-1}} = (1+g_{A_{E,t}})\frac{\tau p_{E,t}}{A_{E,t}} \leq (1+\eta_E)\frac{\tau p_{E,t}}{A_{E,t}}$. Putting everything together, $\left(\frac{\tau_t p_{E,t}}{A_{E,t-1}}\right)^{\frac{1}{2}} (1+\eta_E)^{\frac{1}{2}} \leq (1+\eta_E)\left(\frac{\tau p_{E,t}}{A_{E,t}}\right)^{\frac{1}{2}} < 1+\eta_E$.

B.6 Dynamical System

In this section, I show that the dynamics of the model economy are captured by a discrete dynamical system with four state variables and one control variable. I consider the case of a competitive equilibrium with environmental policy as defined in Section 3.5. A laissez-faire equilibrium is the special case where $\tau_0 = 1$ and $g_\tau = \eta^S = 0$.

B.6.1 Key Equations

I start by discussing an expanded system that highlights the intuition for the results and choice of intensive form variables.[47] The first state variable is the rate of energy taxation, which grows exogenously,

$$\tau_{t+1} = (1 + g_\tau)\tau_t. \tag{B.41}$$

The capital accumulation equation and Euler equation are identical to those in the standard neoclassical growth model. As a result, it helps to define $A_t \equiv TFP_t^{\frac{1}{1-\alpha}} = A_{N,t}\left[1 - \frac{p_{E,t}}{A_{E,t}}\right]^{\frac{1}{1-\alpha}}$ and $g_t \equiv \frac{A_t - A_{t-1}}{A_{t-1}}$. Now, let $c_t = C_t/(A_t L_t)$, which is the control variable, and $k_t = K_t/(A_t L_t)$, which is the second state variable.

The productivity terms are endogenous. As a result, I take $\tilde{p}_t \equiv \frac{\tau_t p_{E,t}}{A_{E,t-1}}$, to be the third state variable. As shown in (20) and (21), $\tilde{p}_t$ determines the growth rate of the two technology terms. In particular,

$$g_{A_E,t} = \eta_E \Gamma(\tilde{p}_t)^{1-\lambda} \equiv g_{A_E}(\tilde{p}_t), \tag{B.42}$$

where $\Gamma(\cdot)$ is the implicit function for $R_{E,t}$ examined in Appendix Section B.5.2. The market clearing condition for R&D resources, (9), is the same in all periods. Using the law of motion for technology, (8), it can be expressed in terms of technology growth rates:

$$g_{A_{N,t}} = \eta_N \left[1 - \left(\frac{g_{A_E}(\tilde{p}_t)}{\eta_E}\right)^{\frac{1}{1-\lambda}}\right]^{1-\lambda} \equiv g_{A_N}(\tilde{p}_t). \tag{B.43}$$

In addition, it is helpful to note that $\frac{p_{E,t}}{A_{E,t}} = (1 + g_{A_E}(\tilde{p}_t))^{-1} \cdot \tau_t^{-1} \cdot \tilde{p}_t$. Putting everything together, the growth rate of A_t is a function of τ_{t-1}, $\tilde{p}_t$ and $\tilde{p}_{t-1}$:

$$g_t = (1 + g_{A_N}(\tilde{p}_t))\left(\frac{1 - (1 + g_{A_E}(\tilde{p}_t))^{-1}(1 + g_\tau)^{-1}\tau_{t-1}^{-1}\tilde{p}_t}{1 - (1 + g_{A_E}(\tilde{p}_{t-1}))^{-1}\tau_{t-1}^{-1}\tilde{p}_{t-1}}\right)^{\frac{1}{1-\alpha}} - 1 \equiv g(\tau_{t-1}, \tilde{p}_t, \tilde{p}_{t-1}). \tag{B.44}$$

[47] It is straightforward to rewrite the system defining $k_t = \frac{K_t}{A_{t-1}L_t}$ and $e_t = \frac{K_t^\alpha (A_{N,t-1}L_t)^\alpha}{A_{E,t-1}E_{t-1}}$ to highlight that both of these variables are predetermined. This complicates the exposition without adding new insight.

Now, it is straightforward to derive standard equations from the neoclassical growth model. From (7), (10), and (13), the law of motion for capital is given by

$$k_{t+1} \quad = \quad \frac{k_t^\alpha - c_t}{(1+g(\tau_t, \tilde{p}_{t+1}, \tilde{p}_t)) \cdot (1+n)}. \tag{B.45}$$

From (22) and (24), the Euler equation is

$$c_{t+1}^\sigma \quad = \quad \beta\alpha \left(\frac{1-(1+g_{A_E}(\tilde{p}_{t+1}))^{-1} \cdot \tilde{p}_{t+1}}{1-(1+g_{A_E}(\tilde{p}_{t+1}))^{-1}\tau_{t+1}^{-1} \cdot \tilde{p}_{t+1}} \right) \left(\frac{k_{t+1}^{\alpha-1}}{(1+g(\tau_t, \tilde{p}_{t+1}, \tilde{p}_t))^\sigma} \right) c_t^\sigma, \tag{B.46}$$

where $r_{t+1} = \alpha \left(\frac{1-(1+g_{A_E}(\tilde{p}_{t+1}))^{-1} \cdot \tilde{p}_{t+1}}{1-(1+g_{A_E}(\tilde{p}_{t+1}))^{-1}\tau_{t+1}^{-1} \cdot \tilde{p}_{t+1}} \right) k_{t+1}^{\alpha-1} = \alpha \frac{\left(1-\frac{\tau_{t+1} p_{E,t+1}}{A_{E,t+1}}\right)}{\left(1-\frac{p_{E,t+1}}{A_{E,t+1}}\right)} k_{t+1}^{\alpha-1} = \alpha \tilde{\tau}_{t+1} k_{t+1}^{\alpha-1}$ is the real interest rate at time $t+1$.

It is also necessary to capture the change in energy extraction costs, which will in turn affect the evolution of R&D incentives. The final state variable is $e_t \equiv \frac{E_t}{\bar{E}_{t-1}}$, the ratio of period-$t$ flow energy use to the stock of energy used prior to the beginning of period t. The dynamics are given by

$$e_t = (1+g_{E,t}) \cdot \frac{e_{t-1}}{1+e_{t-1}}, \tag{B.47}$$

where $(1+g_{E,t}) \equiv E_t / E_{t-1}$ is the growth factor of flow energy use. At any point in time, energy use is given by

$$E_t = \frac{K_t^\alpha \left(A_{N,t} L_t\right)^{1-\alpha}}{A_{E,t}}.$$

As a result,

$$1+g_{E,t} = (k_t/k_{t-1})^\alpha \cdot \frac{(1+g(\tau_{t-1}, \tilde{p}_t, \tilde{p}_{t-1}))^\alpha (1+g_{A_N}(\tilde{p}_t))^{1-\alpha}}{1+g_{A_E}(\tilde{p}_t)} \cdot (1+n) \tag{B.48}$$

and

$$e_t = (k_t/k_{t-1})^\alpha \cdot \frac{(1+g(\tau_{t-1}, \tilde{p}_t, \tilde{p}_{t-1}))^\alpha (1+g_{A_N}(\tilde{p}_t))^{1-\alpha}}{1+g_{A_E}(\tilde{p}_t)} \cdot (1+n) \cdot \frac{e_{t-1}}{1+e_{t-1}}. \tag{B.49}$$

From (4), the pre-tax growth factor of energy prices is given by

$$1+g_{p,t} \equiv p_{E,t}/p_{E,t-1} = (1-g_{A_V})(1+e_{t-1})^\psi. \tag{B.50}$$

It is now possible to specify a law of motion that gives $\tilde{p}_t$ as a function of $\tilde{p}_{t-1}$ and e_{t-1}:

$$\tilde{p}_t = \frac{(1+g_\tau)(1-g_{A_V})(1+e_{t-1})^{\psi}}{(1+g_{A_E}(\tilde{p}_{t-1}))}\tilde{p}_{t-1} \equiv \hat{p}\left(\tilde{p}_{t-1}, e_{t-1}\right). \tag{B.51}$$

B.6.2 Five Variable System

Equations (B.41) – (B.51) describe the period-to-period dynamics of the economy in a form that is convenient for the analysis of the BGP and the computational solution of the model (see Appendix Sections B.6.3, B.7.1, and B.8.1). For theoretical completeness and the stability analysis, I now condense the system down to five variables and five boundary conditions.

Equation (B.41) captures the evolution of τ_t, and equation (B.51) captures the evolution of $\tilde{p}_t$. Combining (B.51) with (B.44) also gives g_{t+1} as a function of e_t, $\tilde{p}_t$ and τ_t, which I will label $g_{t+1} = \hat{g}(\tau_t, \tilde{p}_t, e_t)$. Plugging this result into (B.45) gives

$$k_{t+1} = \frac{k_t^{\alpha} - c_t}{(1+\hat{g}(\tau_t, \tilde{p}_t, e_t)) \cdot (1+n)} \equiv \hat{k}(\tau_t, k_t, c_t, \tilde{p}_t, e_t). \tag{B.52}$$

Next, plugging (B.51), (B.52), and $\hat{g}(\tau_t, \tilde{p}_t, e_t)$ into (B.46) gives

$$\begin{aligned} c_{t+1} &= \left(\beta\alpha\left(\frac{1-(1+g_{A_E}(\hat{p}(\tilde{p}_t, e_t)))^{-1}\cdot\hat{p}(\tilde{p}_t, e_t)}{1-(1+g_{A_E}(\hat{p}(\tilde{p}_t, e_t))^{-1}(1+g_\tau)^{-1}\tau_t^{-1}\cdot\hat{p}(\tilde{p}_t, e_t)}\right)\left(\frac{\hat{k}(\tau_t, k_t, c_t, \tilde{p}_t, e_t)^{\alpha-1}}{(1+\hat{g}(\tau_t, \tilde{p}_t, e_t))^{\sigma}}\right)c_t^{\sigma}\right)^{\frac{1}{\sigma}} \\ &\equiv \hat{c}(\tau_t, k_t, c_t, \tilde{p}_t, e_t). \end{aligned} \tag{B.53}$$

Finally, plugging the above results into (B.49) gives

$$\begin{aligned} e_{t+1} &= (\hat{k}(\tau_t, k_t, c_t, \tilde{p}_t, e_t)/k_t)^{\alpha} \cdot \frac{(1+\hat{g}(\tau_t, \tilde{p}_t, e_t))^{\alpha}(1+g_{A_N}(\hat{p}(\tilde{p}_t, e_t))^{1-\alpha}}{1+g_{A_E}(\hat{p}(\tilde{p}_t, e_t))} \cdot (1+n) \cdot \frac{e_t}{1+e_t} \\ &\equiv \hat{e}(\tau_t, k_t, c_t, \tilde{p}_t, e_t). \end{aligned} \tag{B.54}$$

Equations (B.41) and (B.51) – (B.54) define a mapping $\{\tau_t, k_t, c_t, \tilde{p}_t, e_t\} \to \{\tau_{t+1}, k_{t+1}, c_{t+1}, \tilde{p}_{t+1}, e_{t+1}\}$.

Next, I turn to the boundary conditions. First, τ_0 is given. As explained in Section 3.3, the variables $A_{J,-1}$ for $J = E, N, V$, as well as K_0, L_0, and $\bar{E}_{-1}$, are also given. This implies that

$$\tilde{p}_0 = \frac{p_{E,0}}{A_{E,-1}} \tag{B.55}$$

is given. This, in turn, implies that $A_{E,0} = (1+g_{A_E}(\tilde{p}_0))A_{E,-1}$, $A_{N,0} = (1+g_{A_N}(\tilde{p}_0))A_{N,-1}$, and $A_0 = A_{N,0}\left[1 - \frac{p_{E,0}}{A_{E,0}}\right]^{\frac{1}{1-\alpha}}$ are given. With these results, we also have the following two initial conditions,

$$k_0 = \frac{K_0}{A_0 L_0}, \tag{B.56}$$

$$e_0 = \frac{E_0}{\bar{E}_{-1}} = \frac{K_0^{\alpha}\left(A_{N,0}L_0\right)^{1-\alpha}}{A_{E,0}\bar{E}_{-1}}. \tag{B.57}$$

Finally, the terminal condition for the representative household's maximization problem (23) holds if and only if[48]

$$\lim_{T\to\infty}\left(\beta(1+n)\right)^T \cdot \left(\prod_{t=1}^{T}\left(1+g(\tau_{t-1},\tilde{p}_{t-1},e_{t-1})\right)^{1-\sigma}\right)\cdot\left(1+g(\tau_t,\tilde{p}_t,e_t)\right)\cdot c_T^{-\sigma}k_{T+1} = 0. \tag{B.58}$$

Together, the five equations, (B.41) and (B.51) – (B.54), and five boundary conditions, τ_0 and (B.55) – (B.58), fully characterize the evolution of the competitive equilibrium.

B.6.3 Computational Solution

For the comparison to the Cobb-Douglas model in Section 5.1.1 and the rebound analyses in Section 5.2.1, I solve the decentralized version of the DTC model using the equations presented in Appendix Section B.6.1 and *Dynare* (Adjemian et al., 2011). Since the model solves quickly, I include intermediate equations for y_t, r_t, and $p_{E,t}/A_{E,t}$, which makes the code easier to read and adapt to other purposes. I also generalize the model to allow τ_t to grow at a non-constant rate (as in Section 5.1.1) and to allow for a single-period unexpected shock to $A_{E,t}$ (as in Section 5.2.1). In all of the analyses, the economy is calibrated to the LFBGP and is unexpectedly pushed away from the LFBGP by policy or a technology shock. For the period in which the economy is pushed away from the LFBGP, it is necessary to distinguish between predetermined and jump variables. So, I also update the model to distinguish between savings per effective worker $s_{t-1} \equiv \frac{Y_{t-1}-C_{t-1}}{A_{t-1}L_{t-1}}$, which is predetermined, and capital per effective worker $k_t = \frac{Y_{t-1}-C_{t-1}}{A_t L_t}$, which is affected by contemporaneous changes in technology.

[48]Using the intensive form variables, the terminal condition can be rewritten as

$$A_0 L_0 \lim_{T\to\infty}\left(\beta(1+n)\right)^T \cdot \left(\prod_{t=1}^{T}(1+g_t)^{1-\sigma}\right)\cdot(1+g_{T+1})\cdot c_T^{-\sigma}k_{T+1} = 0.$$

Using (B.44) and dividing by $A_0 L_0$ gives the result.

B.7 Laissez-Faire Balanced Growth Path

B.7.1 Proof of Proposition 1

In this section, I prove Proposition 1, which characterizes the laissez-faire BGP (LFBGP). This section uses the intensive form of the dynamical system introduced in Appendix Section B.6. I re-use equation labels from the main text where possible and reproduce Figure 4. The two key assumptions for this section are

$$(1 - g_{A_V}) \in \left(\frac{1}{[(1+n)(1+\eta_N)]^{\psi}}, \frac{(1+\eta_E)^{1+\psi}}{(1+n)^{\psi}} \right). \tag{A1}$$

and

$$1 - g_{A_V} < (1 + \underline{g_{A_N}})(1+n). \tag{A2}$$

I start by proving several helpful lemmas.

Lemma B.1. *Consider a dynamic path along which the intensive form variables, $\{\tau_t, k_t, c_t, \tilde{p}_t, e_t, \}$, are all constant and the boundary conditions are satisfied. In this case, the economy is on a BGP.*

Proof. Appendix Section B.6 shows that the competitive equilibrium of the economy is captured by the five dynamic equations (B.41) and (B.51) – (B.54) and the five boundary conditions, τ_0 and (B.55) – (B.58). It remains to be shown that if $\{\tau_t, k_t, c_t, \tilde{p}_t, e_t\}$ are all constant, then the economy is on a BGP as defined in Definition 4.

From (B.42) and (B.43), a constant $\tilde{p}_t$ implies that the technology growth rates, $g_{A_E,t}$ and $g_{A_N,t}$ are constant. In addition, if τ_t is constant, then g_t is constant from (B.44). Now, $K_t = A_t L_t k_t$ and $C_t = A_t L_t c_t$ must grow at constant rates. Also, $Y_t = K_t^{\alpha} (A_t L_t)^{1-\alpha}$, which grows at a constant rate. The growth rate of energy use is given by (B.48), which is constant. □

Lemma B.2. *If the economy is on a LFBGP, then the intensive form variables, $\{\tau_t, k_t, c_t, \tilde{p}_t, e_t\}$, are constant.*

Proof. By the definition of laissez-faire, $\tau_t = 1\ \forall t$ on a LFBGP. Also, by Definition 4, the technology growth rates are constant on a BGP. Now, from (B.42), $\tilde{p}_t$ must be constant on a LFBGP. From (B.51), this implies that e_t is constant on a LFBGP. Since $\tau_t = 1\ \forall t$ and $\tilde{p}_t$ is constant, equation (B.44) implies that A_t grows at rate $g^* = g^*_{A_N}$. By definition of a BGP, C_t grows at a constant rate. Since C_t, A_t, and L_t all grow at constant rates, c_{t+1}/c_t must be constant. Thus, equation (B.46) implies that k_t is constant. Finally, equation (B.45) implies that c_t must be constant. $\square$

Lemma B.3. *Consider a LFBGP with increasing energy use along which Assumption (A1) holds. The unique technology growth rates, $\{g^*_{A_E}, g^*_{A_N}\}$, are determined by the following two equations:*

$$g^*_{A_E} = \eta_E \left[1 - \left(\frac{g^*_{A_N}}{\eta_N}\right)^{\frac{1}{1-\lambda}}\right]^{1-\lambda}, \quad g^*_{A_N} \in [0, \eta_N], \tag{RD-MC}$$

and

$$g^*_{A_E} = \left((1 - g_{A_V})(1 + g^*_{A_N})^{\psi}(1+n)^{\psi}\right)^{\frac{1}{1+\psi}} - 1. \tag{RD-BGP}$$

Proof. To start, rewrite the market clearing condition for R&D inputs, (B.43), in terms of g_{A_E} and evaluate on the BGP. This gives

$$g^*_{A_E} = \eta_E \left[1 - \left(\frac{g^*_{A_N}}{\eta_N}\right)^{\frac{1}{1-\lambda}}\right]^{1-\lambda}, \quad g^*_{A_N} \in [0, \eta_N]. \tag{RD-MC}$$

From Lemma B.2, $k_t = k^*$ and $g^* = g^*_{A_N}$ are constant. Plugging these results into (B.49) gives the growth rate of flow and cumulative energy use,

$$1 + e^* = 1 + g^*_E = \frac{1 + g^*_{A_N}}{1 + g^*_{A_E}}(1+n), \tag{B.59}$$

which is assumed to be greater than zero. Plugging (B.59) into (B.50), the growth factor of energy prices on the LFBGP is given by

$$1 + g^*_P = (1 - g_{A_V})\left(\frac{(1 + g^*_{A_N})(1+n)}{1 + g^*_{A_E}}\right)^{\psi}. \tag{B.60}$$

Since $\tilde{p}_t$ and τ_t are constant, $g^*_p = g^*_{A_E}$. Plugging this result into (B.60) and rearranging yields,

$$g^*_{A_E} = \left((1 - g_{A_V})(1 + g^*_{A_N})^{\psi}(1+n)^{\psi}\right)^{\frac{1}{1+\psi}} - 1. \tag{RD-BGP}$$

I have shown that the two equations (RD-MC) and (RD-BGP) must hold on a LFBGP with increasing energy use. I now show that these two equations have a unique solution, as long as Assumption (A1) holds. Equation (RD-MC) describes a continuous, downward sloping relationship between $g^*_{A_E}$ and $g^*_{A_N}$ in (g_{A_N}, g_{A_E}) space, and (RD-BGP) describes a

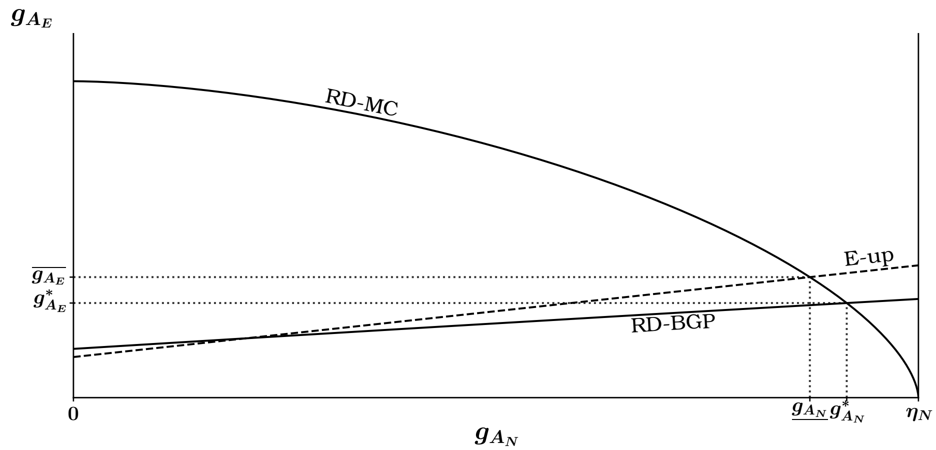


Figure B.2: Reproduction of Figure 4. Technology growth rates on the laissez-faire BGP.

continuous, upward sloping relationship. Thus, there is at most one pair of BGP growth rates that solve these two equations. These two curves are depicted in Figure B.2, which is the same as Figure 4 in the main text. They intersect if and only if $\left[(1 - g_{A_V})(1 + n)^{\psi}\right]^{\frac{1}{1+\psi}} - 1 < \eta E$ and $\left[(1 - g_{A_V})(1 + n)^{\psi}(1 + \eta_N)^{\psi}\right]^{\frac{1}{1+\psi}} - 1 > 0$. These are the conditions imposed by Assumption (A1). □

Before continuing, it is worth examining the intuition behind Assumption (A1). The lower bound ensures that the relationship (RD-BGP) is feasible if $g^*_{A_N} = 0$. If this condition did not hold, the growth rate of energy efficiency would be unable to keep up with the growth rate of energy prices, even if all R&D effort was devoted to energy efficiency. This could only occur if the exogenous component of extraction productivity was sufficiently slow (i.e., g_{A_V} was sufficiently far below zero). The upper bound of Assumption (A1) ensures that the relationship (RD-BGP) is feasible if $g^*_{A_N} = \eta_N$. In other words, it ensures that the growth rate of energy prices would be positive if no R&D resources were devoted to energy efficiency.

Lemma B.4. *Consider a LFBGP with increasing energy use along which Assumption (A1) holds. The unique values of the intensive form variables, $\{\tau_t, k^*, c^*, \tilde{p}^*, e^*\}$, are determined* *by the following equations:*

$$
\begin{aligned}
k^* &= \left(\frac{\alpha\beta}{1+g^*_{A_N}}\right)^{\frac{1}{1-\alpha}}, && \text{(B.61)}\\
c^* &= (k^*)^\alpha - (k^*)(1+g^*_{A_N})(1+n), && \text{(B.62)}\\
e^* &= 1+g^*_E = \frac{1+g^*_{A_N}}{1+g^*_{A_E}}(1+n)-1, && \text{(B.63)}\\
\tilde{p}^* &= \Gamma^{-1}\left(\left[\frac{g^*_{A_E}}{\eta_E}\right]^{\frac{1}{1-\lambda}}\right), && \text{(B.64)}\\
\tau^* &= 1, && \text{(B.65)}
\end{aligned}
$$

where $\{g^*_{A_N}, g^*_{A_E}\}$ *are the values described in Lemma B.3 and* $\Gamma(\cdot)$ *is the monotonic function discussed in Appendix Section B.5.2.*

Proof. Lemma B.2 rules out the existence of a LFBGP represented by a time-varying set of intensive form variables. So, a LFBGP with increasing energy use must be represented by a constant set $\{\tau_t, k^*, c^*, \tilde{p}^*, e^*\}$. Lemma B.3 identifies the unique pair of technology growth rates, $\{g^*_{A_E}, g^*_{A_N}\}$, for a LFBGP with increasing energy use. I now derive the expressions for $\{\tau_t, k^*, c^*, \tilde{p}^*, e^*\}$, conditional on the values of $\{g^*_{A_E}, g^*_{A_N}\}$.

By the definition of laissez-faire, $\tau_t = 1\ \forall t$. On a LFBGP, aggregate productivity, A_t, grows at rate $g^* = g^*_{A_N}$. Plugging these results into (B.46) and rearranging yields (B.61). Combining this result with (B.45) yields (B.62). Also, (B.63) is given by plugging $k = k^*$ and $g^* = g^*_{A_N}$ into (B.49). Finally, (B.64) follows from inverting (B.42). □

Lemma B.5. *For a sufficiently low* β*, the technology growth rates* $\{g^*_{A_E}, g^*_{A_N}\}$ *identified in Lemma B.3 and the set of intensive for variables* $\{\tau_t, k^*, c^*, \tilde{p}^*, e^*\}$ *identified in Lemma B.4 are consistent with the boundary conditions for a competitive equilibrium:* τ_0 *and (B.55) – (B.58).*

Proof. The initial conditions are trivially satisfied. It remains to be shown that the resulting system is consistent with the terminal condition. Evaluated on the LFBGP, (B.58) becomes

$$
\lim_{T\to\infty} (\beta(1+n))^T \left(\prod_{t=0}^{T}(1+g^*_{A_N})^{1-\sigma}\right)(1+g^*_{A_N})(c^*)^{-\sigma}k^* = 0, \tag{B.66}
$$

which takes advantage of the fact that $g^* = g^*_{A_N}$. This holds true if and only if

$$\beta(1+n)(1+g^*_{A_N})^{1-\sigma} < 1.$$

From Lemma B.3, it is clear that the $g^*_{A_N}$ does not depend on β. Also, $g^*_{A_N}$ is bounded above by η_N. Thus, the terminal condition will always be satisfied as long as β is sufficiently low. □

Lemma B.6. *If Assumptions (A1) and (A2) hold, then there is a LFBGP where the growth rate of energy use is positive, the growth rates of technology are given in Lemma B.3, and the values of the intensive form variables are given in Lemma B.4.*

Proof. Lemmas (B.3) and (B.4) identify unique values of technology growth rates and intensive form variables that must hold on a LFBGP with a positive growth rate of energy use. This implies that there is at most one LFBGP with increasing energy use. Lemmas (B.1) and (B.5) further indicate that the economy must be on a LFBGP if the conditions in Lemmas (B.3) and (B.4) hold. It remains to be shown that the LFBGP defined by Lemmas (B.3) and (B.4) would actually have a positive growth rate of energy use. Here, I show that this is guaranteed by Assumption (A2). Figure B.2 depicts the results.

From equation (B.63) in Lemma B.4, the growth rate of energy use is positive on a LFBGP if and only if $g^*_{A_E} < (1+g^*_{A_N})(1+n) - 1$. Consider the cutoff points where the growth rate is exactly zero:

$$g^*_{A_E} = (1+g^*_{A_N})(1+n) - 1. \tag{E-up}$$

Define $(\underline{g_{A_N}}, \overline{g_{A_E}})$ as the solution to (E-up) and (RD-MC). This is the point where all R&D inputs are being used and the growth rate of energy use is zero. The BGP growth rate of energy use is positive if and only if $g^*_{A_N} > \underline{g_{A_N}}$ (equivalently, $g^*_{A_E} < \overline{g_{A_E}}$). A necessary and sufficient condition for this to hold is that

$$(1+\underline{g_{A_N}})(1+n) - 1 > \left((1-g_{A_V})(1+\underline{g_{A_N}})^{\psi}(1+n)^{\psi}\right)^{\frac{1}{1+\psi}} - 1, \tag{B.67}$$

which is guaranteed by Assumption (A2). The assumption implies that energy efficiency grows faster than energy prices at the point $(\underline{g_{A_N}}, \overline{g_{A_E}})$. Thus, $g^*_{A_N} > \underline{g_{A_N}}$ and $g^*_{A_E} < \overline{g_{A_E}}$ on the LFBGP. This can also be seen geometrically from Figure B.2. The assumption implies that (RD-BGP) lies below (E-up) at point $\underline{g_{A_N}}$. Since (RD-BGP) is strictly increasing and (RD-MC) is strictly decreasing, their intersection must fall to the right of $\underline{g_{A_N}}$. $\square$

Lemma B.7. *If Assumptions (A1) and (A2) hold, then there does not exist a LFBGP with constant or decreasing flow energy use.*

Proof. I assume that flow energy use is constant or decreasing on a LFBGP and show that this generates a contradiction under Assumptions (A1) and (A2). From Definition 4, technology growth rates are constant on a BGP. Given the shape of the R&D production function, (8), R&D allocations are always interior. From equation (20), $p_{E,t}/A_{E,t-1}$ must be constant on a LFBGP, implying that $g_P^* = g_{A_E}^* > 0$.

If flow energy use is constant or decreasing, $e_t = E_t/\bar{E}_{t-1}$ converges to zero and the growth rate of energy prices is given by $(1 + g_P^*) = (1 - g_{A_V})$. Thus,

$$g_{A_E}^* = -g_{A_V}. \tag{RD-BGPalt}$$

The R&D market clearing condition (RD-MC) does not depend on the growth rate of energy use. So, the equilibrium growth rates would be given by the intersection of (RD-BGPalt) and (RD-MC). But, by Assumption (A2), (RD-BGPalt) lies below (E-up) at point $\underline{g_{A_N}}$ in (g_{A_N}, g_{A_E}) space at $\underline{g_{A_N}}$. Thus, $g_{A_N}^* > \underline{g_{A_N}}$ and $g_{A_E}^* < \overline{g_{A_E}}$. As explained in the proof to Lemma B.6, this implies that energy use is increasing on the BGP, which contradicts the assumption that energy use is constant or decreasing. $\square$

The key intuition for the results presented above is summarized in Figure B.2. It is now straightforward to prove Proposition 1, which I reproduce below for reading ease.

Proposition 1 *Let Assumptions (A1) and (A2) hold. For a sufficiently low β, there exists a* unique BGP *in a* laissez-faire equilibrium*. On this BGP, (i) the R&D allocations are implicitly given by*

$$R_E^* = \left\{ \frac{\left[(1 + \eta_N(1 - R_E^*)^{1-\lambda})^{\frac{\alpha}{1-\alpha}}(1+n)(1 - g_{A_V})^{\frac{1}{\psi}}\right]^{\frac{\psi}{1+\psi}} - 1}{\eta_E} \right\}^{\frac{1}{1-\lambda}} \quad \textit{and } R_N^* = 1 - R_E^*;$$

(ii) output, consumption, and the capital stock grow at the same constant factor, $(1+g^*_{A_N})(1+n)$*; (iii) the real interest rate is constant; (iv) flow energy use grows at constant factor* $1+g^*_E = \frac{1+g^*_{A_N}}{1+g^*_{A_E}}(1+n) > 0$*; and (v) the expenditure shares of energy, capital, labor, R&D inputs, and profits are all constant.*

Proof. Lemma B.6 establishes that there is a single LFBGP with increasing energy use. Lemma B.7 establishes that there is no other LFBGP. I will prove each item individually.

- Item (i) follows from plugging the R&D resource constraint, (9), and the law of motion for technology, (8), into (RD-BGP).
- Item (ii) follows from the fact that k_t and c_t are constant on the BGP and the fact that A_t grows at $g^*_{A_N}$ on the BGP. In addition, $Y_t = A_t L_t k_t^{\alpha}$.
- Item (iii) follows from the fact that $r_t = \alpha k_t^{\alpha-1}$.
- Item (iv) follows from equation (B.59) and Lemma B.6.
- For item (v), the expenditure shares of labor, capital, R&D inputs and profits are given by equations, (B.26), (B.28), (B.34), and (B.35), respectively, all of which are constant on a LFGP. As discussed in the main text, $\theta_{E,t} = \frac{p_{E,t}/A_{E,t}}{1-p_{E,t}/A_{E,t}}$, which must be constant on the LFBGP, because $p_{E,t}/A_{E,t} = (1+g_{A_E}(\tilde{p}_t))^{-1}\tilde{p}_t$ is constant.

Since the above results identify the only possible LFBGP growth rates for technologies and macroeconomics aggregates, these growth rates cannot differ based on initial conditions. By Definition 5, therefore, the LFBGP is *unique.* □

B.7.2 Stability

Sections 3.4 and B.7.1 show that the laissez-faire balanced growth path (LFBGP) is consistent with data. In this section, I show that the LFBGP is locally saddle-path stable. The fact that LFBGP is locally stable implies that the economy will converge to the LFBGP, as long as it starts sufficiently nearby. The fact that the LFBGP is saddle-path stable implies that the path of convergence is unique.

Section B.6 shows that the dynamics of laissez-faire competitive equilibrium can be represented by a mapping $\{k_t, c_t, \tilde{p}_t, e_t\} \to \{k_{t+1}, c_{t+1}, \tilde{p}_{t+1}, e_{t+1}\}$, where τ_t is omitted because it is equal to one at all times in a laissez-faire equilibrium. The local stability of the BGP is

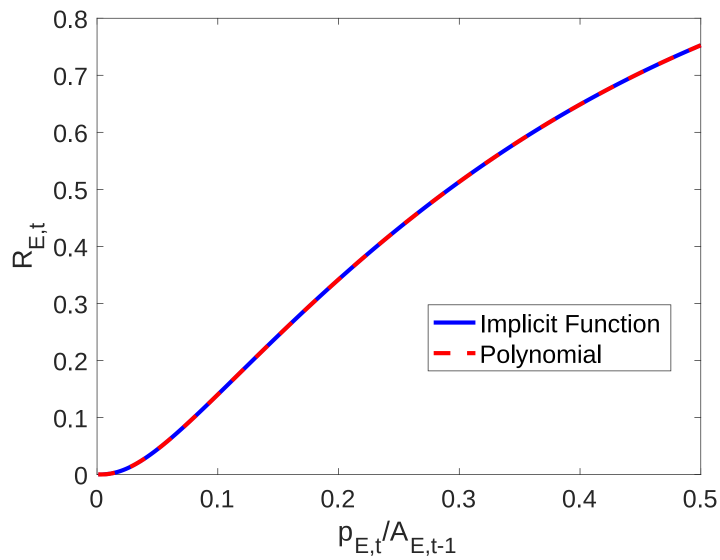


Figure B.3: Approximation of $\Gamma(\frac{p_{E,t}}{A_{E,t-1}})$

determined by the eigenvalues of the Jacobian matrix

$$\begin{pmatrix} a_{kk} & a_{kc} & a_{k\tilde{p}} & a_{ke} \\ a_{ck} & a_{cc} & a_{c\tilde{p}} & a_{ce} \\ a_{\tilde{p}k} & a_{\tilde{p}c} & a_{\tilde{p}\tilde{p}} & a_{\tilde{p}e} \\ a_{ek} & a_{ec} & a_{e\tilde{p}} & a_{ee} \end{pmatrix},$$

where $a_{vw} \equiv \frac{\partial v_{t+1}}{\partial w_t}(k^*, c^*, \tilde{p}^*, e^*)$, for $v, w \in \{k, c, \tilde{p}, e\}$. Since the system has one control variable, c_t, a locally saddle-path stable steady state will have three eigenvalues with (complex) absolute value less than one and one eigenvalue with absolute value greater than one.

Given the size of the system, I have not been able to characterize the eigenvalues analytically. Thus, I calculate the eigenvalues numerically for the baseline parameters and each of the robustness scenarios. As discussed in the context of equation (20) in the main text and Appendix Section B.5.2, the dynamical system depends on the implicit function $R_{E,t} = \Gamma(\frac{p_{E,t}}{A_{E,t-1}})$. To calculate the eigenvalues, I solve the implicit function for 500 values of $\frac{p_{E,t}}{A_{E,t-1}}$ and then approximate the results with a 12th-order polynomial. Figure B.3 shows the fit for the baseline parameters. The LFBGP level for $R_{E,t}$ is 0.1342, which occurs far from the bounds of domain the used to parameterize the polynomial.

The eigenvalues are shown in Table B1. In all cases, the system has three eigenvalues with absolute value less than one and one eigenvalue with an absolute value greater than one, implying that the LFBGP is locally saddle-path stable. When prices are endogenous

Table B1: Stability

Scenario	Eigenvalues	Absolute Value	Local Stability
Baseline	3.00, 0.35	3.00, 0.35	Saddle
	0.87+0.10i, 0.87-0.10i	0.88, 0.88	
$\lambda = 0.21$	3.00, 0.35	3.00, 0.35	Saddle
	0.81+0.08i, 0.81-0.08i	0.81, 0.81	
$\psi = 3.64$	3.00, 0.36	3.00, 0.36	Saddle
	0.85+0.19i, 0.85-0.19i	0.87, 0.87	
$\psi = 0$	3.00, 0.35	3.00, 0.35	Saddle
	0.94, 0.82	0.94, 0.82	

($\psi > 0$), the system has two complex eigenvalues, implying that the economy converges non-monotonically towards the LFBGP. This result is evident in Figure 9, which shows the transition dynamics following a costless technology shock (CTS). Since there is no policy intervention, the economy is still in a laissez-faire competitive equilibrium after the CTS. In Figure 9, both energy use and consumption overshoot their long-run levels as they transition back to a LFBGP after the initial shock to energy efficiency. When energy prices are exogenous ($\psi = 0$), there are no complex eigenvalues, implying that the economy transitions back to a LFBGP monotonically. Figure D.18 demonstrates that there is monotonic convergence after a cost-less technology shock when energy prices are exogenous.

B.8 Environmental Policy

B.8.1 Balanced Growth with Environmental Policy

In this section, I prove Proposition 2, which characterizes the BGP with environmental policy (EPBGP). This section uses the intensive form of the dynamical system introduced in Appendix Section B.6. I re-use equation labels from the main text where possible. Many of the results closely follow those in Section B.7.1. I focus on the case where $g_\tau > 0$. If $g_\tau = 0$, the analysis is identical to the laissez-faire case. The two key assumptions for this section are:

$$(1 + g_\tau) < \frac{1 + \eta_E}{1 - g_{A_V}} \tag{A3}$$

and

$$(1 + g_\tau) \geq \frac{1 + \underline{g_{A_E}}}{1 - g_{A_V}}. \tag{A4}$$

Lemma B.8. *Consider a dynamic path along which $\tau_t \to \infty$, the intensive form variables $\{k_t, c_t, \tilde{p}_t, e_t,\}$ are all constant, and the boundary conditions, τ_0 and (B.55) – (B.58), are satisfied. In this case, the economy converges to a BGP.*

Proof. Appendix Section B.6 shows that the competitive equilibrium of the economy is captured by the five dynamic equations, (B.41) and (B.51) – (B.54), and the five boundary conditions, τ_0 and (B.55) – (B.58). It remains to be shown that if $\tau_t \to \infty$ and the intensive form variables $\{k_t, c_t, \tilde{p}_t, e_t\}$ are all constant, then the economy converges to a BGP as defined in Definition 4.

First, it will help to simplify the dynamical system. Since constant $\tilde{p}_t \equiv \frac{\tau_t p_{E,t}}{A_{E,t-1}}$ is constant, $\frac{p_{E,t}}{A_{E,t}} \to 0$ as $\tau_t \to \infty$. Thus, along the assumed path (B.44) converges to

$$g_t = g_{A_N}(\tilde{p}^*). \tag{B.68}$$

and (B.46) converges to

$$(c^*)^\sigma = \beta\alpha \left(1 - (1 + g_{A_E}(\tilde{p}^*))^{-1} \cdot \tilde{p}^*\right) \left(\frac{(k^*)^{\alpha-1}}{(1 + g(\tau_t, \tilde{p}^*, \tilde{p}^*))^\sigma}\right) (c^*)^\sigma . \tag{B.69}$$

From (B.42) and (B.43), a constant $\tilde{p}_t$ implies that the growth rates of each technology, $g_{A_E,t}$ and $g_{A_E,t}$, are constant. So, g_t converges to a constant as shown in (B.68). Now, $K_t = A_t L_t k_t$ and $C_t = A_t L_t c_t$ must grow at constant rates. Also, $Y_t = K_t^\alpha (A_t L_t)^{1-\alpha}$, which grows at a constant rate. Since K_t and the technology growth rates are constant, equation (B.48) shows that the growth rate of energy use, g_E^*, is also constant. □

Lemma B.9. *The intensive form variables, $\{k_t, c_t, \tilde{p}_t, e_t\}$, are constant on any BGP with environmental policy.*

Proof. By definition the technology growth rates are constant on a BGP. From (B.42), $\tilde{p}_t$ must be constant on a BGP. From (B.51), this implies that e_t is constant on a BGP. By definition, $e_t \geq 0$. From (B.47), a constant e_t can occur in two ways. Each is only possible asymptotically. First, $e^* = 0$. This will occur if the growth of flow energy use is (weakly) negative in the limit. Second, $e^* = g_E^* > 0$. If flow energy use grows at a positive constant rate forever, then the cumulative stock of energy will grow at the same rate in the limit. Equation (B.68) implies that $g^* = g_{A_N}^*$. By definition, C_t grows at a constant rate on the BGP. Since C_t, A_t, and L_t all grow at constant rates, c_{t+1}/c_t is constant on a BGP. Thus, equation (B.46) implies that k_t is constant on the BGP. Finally, equation (B.45) implies that c_t must be constant on a BGP. $\square$

Lemma B.10. *Let Assumptions (A1) – (A3) hold. On an EPBGP, the technology growth rates $\{g^*_{A_E}, g^*_{A_N}\}$ are unique. If flow energy use is increasing ($g^*_E > 0$), then these unique growth rates solve:*

$$g^*_{A_E} = \eta_E \left[1 - \left(\frac{g^*_{A_N}}{\eta_N}\right)^{\frac{1}{1-\lambda}}\right]^{1-\lambda}, \quad g^*_{A_N} \in [0, \eta_N], \tag{RD-MC}$$

and

$$g^*_{A_E} = \left((1 - g_{A_V})(1 + g_\tau)(1 + g^*_{A_N})^\psi (1 + n)^\psi\right)^{\frac{1}{1+\psi}} - 1. \tag{RD-BGP$'$}$$

If $g^*_E \leq 0$, then the growth rates solve (RD-MC) and

$$g^*_{A_E} = (1 - g_{A_V})(1 + g_\tau) - 1. \tag{RD-BGP$''$}$$

Proof. The market clearing condition for R&D resources, (RD-MC), is unaffected by the introduction of environmental policy and is therefore the same as in Lemma B.3. Now, I consider two cases separately.

- Case 1: Assume $g^*_E > 0$. Following the steps from the proof of Lemma B.3 yields

$$g^*_{A_E} = \left((1 - g_{A_V})(1 + g_\tau)(1 + g^*_{A_N})^\psi (1 + n)^\psi\right)^{\frac{1}{1+\psi}} - 1. \tag{RD-BGP$'$}$$

  This is nearly identical to the laissez-faire case from Lemma B.3, except that R&D incentives depend on tax-inclusive energy prices. Equation (RD-BGP$'$) describes an upward sloping relationship in (g_{A_N}, g_{A_E}) space. Equation (RD-MC) describes a downward sloping relationship. This situation is depicted in Figure B.4. Noting the possible range for growth rates, the two lines cross exactly once if and only if

$$\left((1 - g_{A_V})(1 + g_\tau)(1 + n)^\psi\right)^{\frac{1}{1+\psi}} - 1 < \eta_E$$

  and

$$\left((1 - g_{A_V})(1 + g_\tau)(1 + \eta_N)^\psi (1 + n)^\psi\right)^{\frac{1}{1+\psi}} - 1 > 0.$$

  The first condition is guaranteed by Assumption (A3) and the second is guaranteed by

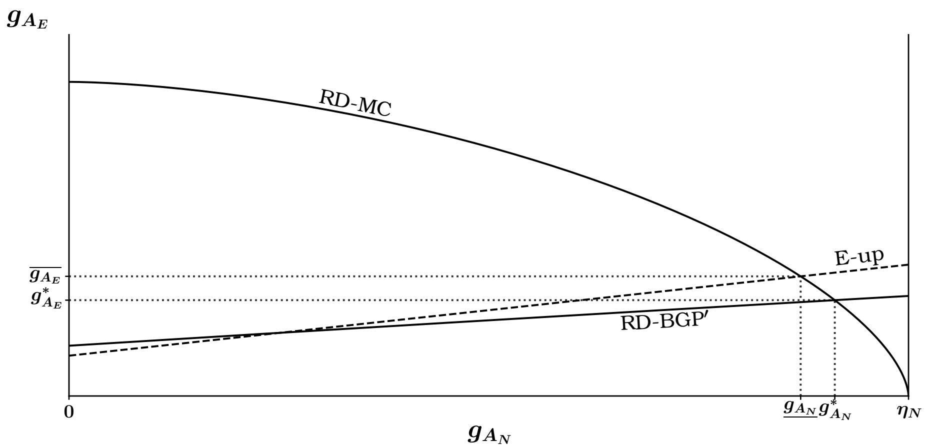


Figure B.4: Technology growth rates with environmental policy and positive growth in energy use.

Assumption (A1).

- Case 2: Assume $g^*_E \leq 0$, which implies that $e^* = 0$. From (B.51),

$$g^*_{A_E} = (1 - g_{A_V})(1 + g_\tau) - 1. \tag{RD-BGP''}$$

As before, growth in energy efficiency is just rapid enough to offset increasing tax-inclusive prices. Since the growth rate of $\bar{E}_t$ converges to zero, the growth in energy prices is driven entirely by the exogenous forces of extraction technology and the rate of taxation.

This outcome is depicted in Figure B.5, which is a reproduction of Figure 5 from the main text. Equation (RD-BGP″) is a horizontal line in (g_{A_N}, g_{A_E}) space. It intersects (RD-MC) exactly once, as long as $(1 - g_{A_V})(1 + g_\tau) \in (0, 1 + \eta_E)$, which is guaranteed by Assumptions (A1) and (A3) in conjunction with the restriction that $g_\tau \geq 0$.

□

Lemma B.11. *Let Assumptions (A1)–(A3) hold. On a BGP with environmental policy, the values of the intensive form variables, $\{k^*, c^*, \tilde{p}^*, e^*\}$, are unique. If flow energy use is increasing ($g^*_E > 0$), then*

$$e^* = \frac{1 + g^*_{A_N}}{1 + g^*_{A_E}}(1 + n) - 1. \tag{B.70}$$

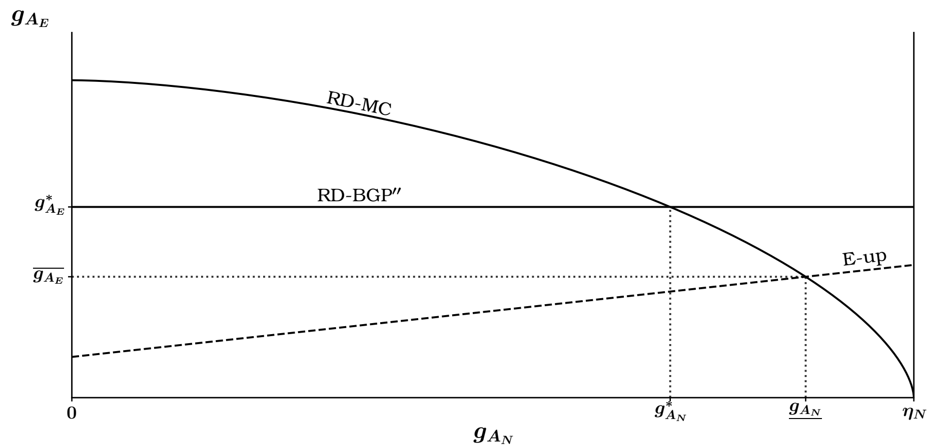


Figure B.5: Reproduction of Figure 5. Technology growth rates with environmental policy and no growth in cumulative energy use.

Otherwise, $e^* = 0$. In either case, the remaining values are given by

$$k^* = \left(\frac{\alpha\beta(1-(1+g^*_{A_E})^{-1}\tilde{p}^*)}{1+g^*_{A_N}}\right)^{\frac{1}{1-\alpha}}, \tag{B.71}$$

$$c^* = (k^*)^\alpha - (k^*)(1+g^*_{A_N})(1+n), \tag{B.72}$$

$$\tilde{p}^* = \Gamma^{-1}\left(\left[\frac{g^*_{A_E}}{\eta_E}\right]^{\frac{1}{1-\lambda}}\right), \tag{B.73}$$

where $\Gamma(\cdot)$ is the monotonic function discussed in Appendix Section B.5.2 and the technology growth rates, $g^*_{A_N}$ and $g^*_{A_E}$, are given in Lemma B.10.

Proof. The previous lemma shows that that technology growth rates are unique, conditional on the sign of g^*_E. I will show that the values of $\{k_t, c_t, \tilde{p}_t, e_t\}$ are unique, conditional on the values of $\{g^*_{A_E}, g^*_{A_N}\}$ and the sign of g^*_E.

If $g^*_E \leq 0$, then the ratio of flow to cumulative energy use, $e_t \equiv E_t/\bar{E}_{t-1}$, must converge to zero, i.e., $e^* = 0$. From Lemma B.9, $k_t = k^*$ and $g^* = g^*_{A_N}$ are constant. If $e^* \geq 0$, plugging these results into (B.49) gives (B.70), which is the same as the laissez-faire condition (B.59). Now, I turn to the other three variables. From (B.68), A_t grows at rate $g^* = g^*_{A_N}$. Conditional on the value for e^*, equation (B.71) follows from (B.69) with constant c_t, (B.72) follows from (B.45) with constant k_t, and (B.73) follows from inverting (B.42). □

Lemma B.12. *For a sufficiently low β, the technology growth rates $\{g^*_{A_E}, g^*_{A_N}\}$ identified in Lemmas B.10 and the set of intensive for variables $\{\tau_t, k^*, c^*, \tilde{p}^*, e^*\}$ defined in Lemma B.11* *are consistent with the boundary conditions for a competitive equilibrium:* τ_0 *and (B.55) – (B.58).*

Proof. The proof of Lemma B.10 shows that $g^*_{A_N}$ is independent of β. Otherwise, the proof of this lemma is identical to that of Lemma B.5. $\square$

Lemma B.13. *If Assumptions (A1) – (A3) hold, then there is a unique EPBGP. In addition, (i) if Assumption (A4) holds, then the EPBGP has constant or decreasing energy use ($g^*_E \leq 0$); (ii) if Assumption (A4) holds with equality, $g^*_E = 0$; and (iii) if Assumption (A4) does not hold, then $g^*_E > 0$.*

Proof. Lemma (B.8) says that the economy is necessarily on a BGP if the intensive form variables are constant and satisfy the boundary conditions. Lemmas (B.10) – (B.12) identify the only two sets of intensive form variables that can define a EPBGP, one that is consistent with increasing energy use and one that is consistent with constant or decreasing energy use. Lemma (B.9) implies that no other EPBGP exists. Here, I show that, for any given set of parameters, there is always exactly one set of intensive form variables that is consistent with an EPBGP, with the sign of g^*_E determined by whether Assumption (A4) holds.

- Case 1: $g^*_E > 0$. From Lemma B.10, equations (RD-BGP′) and (RD-MC) determine the growth growth rates of technology that must hold on an EPBGP with increasing energy use. To have increasing energy use, it must also be the case that $g^*_{A_E} < \overline{g_{A_E}}$. Similar to the laissez-faire case (Lemmas B.6 and B.7), this condition holds if and only if the line defined by (RD-BGP′) lies below the line defined by (E-up) at point $\underline{g_{A_N}}$ in (g_{A_N}, g_{A_E}) space. The condition holds if and only if

$$\left((1-g_{A_V})(1+g_\tau)(1+\underline{g_{A_N}})^\psi(1+n)^\psi\right)^{\frac{1}{1+\psi}} < (1+\underline{g_{A_N}})(1+n) \tag{B.74}$$

$$\iff (1+g_\tau) < \frac{(1+\underline{g_{A_N}})(1+n)}{(1-g_{A_V})} \tag{B.75}$$

$$= \frac{1+\overline{g_{A_E}}}{(1-g_{A_V})}. \tag{B.76}$$

  So, if Assumption (A4) holds, then the growth rates determined by (RD-BGP′) and (RD-MC) are inconsistent with increasing energy use, and there is no BGP with $g^*_E > 0$. If Assumption (A4) does not hold, then the growth rates determined by (RD-BGP′) and (RD-MC) do determine a BGP, and the values of the intensive form variables are given in Lemma B.11.

- Case 2: $g_E^* \leq 0$. From Lemma B.10, $g_E^* \leq 0$ requires that (RD-BGP$''$) lies above (E-up) at point $\underline{g_{A_N}}$ in (g_{A_N}, g_{A_E}) space. This is true if and only if

$$(1 - g_{A_V})(1 + g_\tau) \geq 1 + \overline{g_{A_E}}.$$

So, if Assumption (A4) does not hold, then the growth rates determined by (RD-BGP$''$) and (RD-MC) are inconsistent with (weakly) decreasing energy use, and there is no BGP with $g_E^* \leq 0$. If Assumption (A4) does hold, then the growth rates determined by (RD-BGP$''$) and (RD-MC) do determine a BGP, with the values for the intensive form variables given in Lemma B.11.

In addition, $g_E^* = 0$ if and only if (RD-BGP$''$) and (E-up) intersect at at point $\underline{g_{A_N}}$ in (g_{A_N}, g_{A_E}) space, which happens if and only if $(1 - g_{A_V})(1 + g_\tau) = \overline{g_{A_E}}$.

Putting everything together, there is always a single EPBGP. If Assumption (A4) does not hold, there is a single EPBGP with increasing energy use and no EPBGP with decreasing energy use or constant (condition iii). If Assumption (A4) does hold, then there is a single EPBGP with (weakly) decreasing energy use (condition i). Energy use is exactly zero on the EPBGP if and if only Assumption (A4) holds with equality (condition ii).

$\square$

With Lemmas (B.8) – (B.13), it is straightforward to prove Proposition 2, which is reproduced below.

Proposition 2 *Let Assumptions (A1)–(A3) hold. For a sufficiently low β, there exists a* unique BGP *in an* equilibrium with environmental policy*. Along the BGP, (i) if Assumption (A4) does not hold, R&D allocations are implicitly given by*

$$R_E^* = \left\{ \frac{\left[(1 + \eta_N (1 - R_E^*)^{1-\lambda})^{\frac{\alpha}{1-\alpha}} (1+n) [(1+g_\tau)(1 - g_{A_V})]^{1/\psi} \right]^{\frac{\psi}{1+\psi}} - 1}{\eta_E} \right\}^{\frac{1}{1-\lambda}} \quad \textit{and } R_N^* = 1 - R_E^*,$$

but it Assumption (A4) does hold, R&D allocations are given by

$$R_E^* = \left[\frac{(1 - g_{A_V})(1 + g_\tau) - 1}{\eta_E} \right]^{\frac{1}{1-\lambda}} \quad \textit{and } R_N^* = 1 - R_E^*;$$

(ii) output, consumption, and the capital stock growth at the same constant factor, $(1+$

$g^*_{A_N}$)$(1+n)$; (iii) the real interest rate is constant; (iv) if Assumption (A4) does not hold, flow energy use grows at factor $1+g^*_E = \frac{1+g^*_{A_N}}{1+g^*_{A_E}}(1+n) > 0$, but if Assumption (A4) does hold, $g^*_E \leq 0$; (v) the expenditure shares of energy, capital, labor, R&D inputs, and profits are all constant.

Proof. Lemmas B.10 – B.13 characterize the unique set $\{k^*, c^*, \tilde{p}^*, e^*\}$ that define a EPBGP and how this set depends on Assumption (A4). I will prove each item of the proposition individually.

- If Assumption (A4) does not hold, item (i) follows from plugging the R&D resource constraint, (9), and the law of motion for technology, (8), into (RD-BGP′). If Assumption (A4) does hold, the item follows from plugging these same equations into (RD-BGP″).
- Item (ii) follows from the fact that k_t and c_t are constant on the BGP and the fact that A_t grows at $g^*_{A_N}$ on the BGP. In addition, $Y_t = A_t L_t k_t^{\alpha}$.
- Item (iii) follows from the fact that $r_t = \alpha\tilde{\tau}_t k_t^{\alpha-1}$, where $\tilde{\tau}_t = \frac{1-\frac{\tau_t p_{E,t}}{A_{E,t}}}{1-\frac{p_{E,t}}{A_{E,t}}}$. As $\tau_t \to \infty$, $\tilde{\tau}_t$ converges to $1 - \frac{\tau_t p_{E,t}}{A_{E,t}} = 1 - \tilde{p}_t$, which is constant on the BGP.
- Item (iv) follows from Lemma B.10.
- For item (v), the expenditure shares of labor, capital, R&D inputs, and profits are given by equations (B.26), (B.28), (B.34), and (B.35), respectively, all of which are constant on the BGP. As $\tau_t \to \infty$, $\theta_{E,t} \to \tau_t p_{E,t}/A_{E,t} = (1+g_{A_E,t})^{-1}\tilde{p}_t$, which is constant on a BGP.

For a given set of parameters, the above results identify the only possible EPBGP growth rates for technologies and the macroeconomics aggregates. So, these growth rates cannot differ based on initial conditions. By Definition 5, therefore, the EPBGP is *unique*. □

B.8.2 Least-Cost Path

In this section, I investigate the least-cost path to achieve an exogenous environmental target that is specified in terms of cumulative energy use. For simplicity, I focus on the case of log preferences, as in the quantitative analysis.

Decentralized Equilibrium. — The presentation of the model in Section 3.1 focuses on the case of value-added taxes on energy and subsidies for R&D. This facilitates the examination of balanced growth with environmental policy, taking the policy intervention as given. Here, I focus on per-unit energy taxes (τ_t^u) and R&D subsidies/taxes (η_t^u), which simplifies the analysis of how policy can implement the least-cost path. The competitive equilibrium is the same as in Section 3.2 except that $p_{E,t} + \tau_t^u$ replaces $\tau_t p_{E,t}$ and $p_t^R - \eta_t^u$ replaces $(1-\eta_t^S)p_t^R$. Three updated equations will be important for comparing the decentralized equilibrium to the least-cost path allocations. With per-unit policies, the Euler equation for the decentralized equilibrium becomes

$$C_{t+1} = \alpha\tilde{\beta}\left[1 - \frac{p_{E,t+1} + \tau_{t+1}^u}{A_{E,t+1}}\right] K_{t+1}^{\alpha-1}(A_{N,t+1}L_{t+1})^{1-\alpha}C_t, \tag{EE-DE}$$

where $\tilde{\beta} \equiv \beta(1+n)$ and $r_{t+1} = \alpha\left[1 - \frac{p_{E,t+1}+\tau_{t+1}^u}{A_{E,t+1}}\right] K_{t+1}^{\alpha-1}(A_{N,t+1}L_{t+1})^{1-\alpha}$. The research arbitrage equation in the decentralized equilibrium becomes

$$\frac{p_t^R - \eta_t^u}{p_t^R} = (1-\alpha)^{-1}\frac{\frac{p_{E,t}+\tau_t^u}{A_{E,t}}}{\left[1 - \frac{p_{E,t}+\tau_t^u}{A_{E,t}}\right]}\frac{A_{N,t}}{A_{E,t}}\frac{\eta_E R_{E,t}(i)^{-\lambda}A_{E,t-1}}{\eta_N R_{N,t}(i)^{-\lambda}A_{N,t-1}}. \tag{RA-DE}$$

If $\eta_t^u < 0$, there is a tax on energy efficiency R&D. Since there is a fixed quantity of R&D inputs, policy can influence the direction of R&D but not the overall quantity. As a result, a tax on energy efficiency R&D has the same effect as a subsidy for non-energy R&D. Finally, the price of research inputs in the decentralized equilibrium can be written as

$$p_t^R = \alpha(1-\alpha)(1-\lambda)\left[1 - \frac{p_{E,t} + \tau_t^u}{A_{E,t}}\right] K_t^{\alpha}(A_{N,t}L_t)^{1-\alpha}\frac{\eta_N R_{N,t}^{-\lambda}}{1 + g_{A_N,t}}. \tag{PR-DE}$$

Planning Problem. — All capital goods producers start with the same level of technology. This immediately implies that the social planner will have all capital good producers make identical decisions. As discussed in Appendix Section B.3, this implies that $X_t(i) = K_t$, $R_{J,t}(i) = R_{J,t}$, and $A_{J,t}(i) = A_{J,t}$ $\forall i, J, t$. I focus on aggregate allocations.

The social planner chooses a sequence $\{C_t, K_{t+1}, E_t, A_{N,t}, A_{E,t}, R_{N,t}, R_{E,t}\}_{t=0}^{\infty}$ to maximize the lifetime utility of the representative household (11) subject to the intertemporal budget constraint (13), the Leontief production function (2), Leontief constraint (3), law of motion for energy extraction costs (4), law of motion for endogenous technology (8), and the market clearing condition for R&D inputs (9). In addition, the social planner faces an exogenous environmental target in the form of a constraint on cumulative energy use ($\bar{E}^{\text{target}}$) between periods 0 and T. I refer to the difference between $\bar{E}^{\text{target}}$ and energy use at any time $t < T$ as the remaining 'energy budget'.

A primary goal of the least-cost path analysis is to investigate how the government could implement the optimal allocations through policy. Aside from the environmental target, there are three externalities. First, energy extraction in period t increases extraction costs in future periods. Second, R&D in period t increases the productivity of R&D in future periods. Third, there is a monopoly distortion in the production of capital goods. Throughout the paper, I assume this last distortion is undone via an optimal subsidy for capital good production, including in a laissez-faire equilibrium. As mentioned above, when examining the implementation of environmental policy, I consider the case where the government has access to two instruments, a per-unit tax on energy and a per-unit subsidy/tax on energy efficiency R&D. I also assume that the government has access to lump-sum taxes and transfers, which can be used to balance the budget without affecting incentives.

Analysis. — The Lagrangian for the planning problem is given by

$$\begin{aligned}\mathcal{L} =& L_0 \sum_{t=0}^{\infty} \Bigg[\tilde{\beta}^t \ln C_t - \tilde{\beta}^t \varphi_t \left(C_t + K_{t+1} + A_{V,t}^{-1} \left(\bar{E}_{-1} + \sum_{\hat{t}=0}^{t-1} E_{\hat{t}} \right)^{\psi} E_t - A_{E,t} E_t \right) \\ & - \tilde{\beta}^t \mu_t \left(A_{E,t} E_t - K_t^{\alpha} \left(A_{N,t} L_t \right)^{1-\alpha} \right) \\ & - \tilde{\beta}^t \kappa_t^E \left(A_{E,t} - (1 + \eta_E R_{E,t}^{1-\lambda}) A_{E,t-1} \right) \\ & - \tilde{\beta}^t \kappa_t^N \left(A_{N,t} - (1 + \eta_N R_{N,t}^{1-\lambda}) A_{N,t-1} \right) \\ & - \tilde{\beta}^t \xi_t \left(R_{E,t} + R_{N,t} - 1 \right) \Bigg] \\ & - L_0 \Omega \left(\sum_{\hat{t}=0}^{T} E_{\hat{t}} - \bar{E}^{\text{target}} \right).\end{aligned} \tag{B.77}$$

There are a few important things to note about this formulation. In the first line, the intertemporal budget constraint is written in terms of gross output, rather than final output, and includes the law of motion for energy extraction costs. The second line gives the constraint from Leontief production. Also, for ease of interpretation, the scalar L_0 is factored out of the infinite sum, implying that the multipliers have the form $\beta^t L_t \varphi_t$. Since $\tilde{\beta} \equiv \beta(1+n)$ is the effective discount rate, it is straightforward to interpret φ_t as the current shadow-value of gross output in period t, with a similar interpretation for the other multipliers in the sum. The cumulative energy use constraint has the multiplier $L_0 \Omega$, where Ω is the shadow-value of relaxing the constraint per time-0 person.

I assume that the first order conditions are both necessary and sufficient for an optimum. The first order conditions are:

$$C_t: \quad C_t^{-1} = \varphi_t \tag{B.78}$$

$$K_{t+1}: \quad \tilde{\beta}^{t+1}\mu_{t+1}\alpha K_{t+1}^{\alpha-1}(A_{N,t+1}L_{t+1})^{1-\alpha} = \tilde{\beta}^t\varphi_t \tag{B.79}$$

$$E_{t\leq T}: \quad \tilde{\beta}^t\varphi_t A_{E,t} = \tilde{\beta}^t\mu_t A_{E,t} + \tilde{\beta}^t\varphi_t p_{E,t} + \Omega + \psi\sum_{j=t+1}^{\infty}\tilde{\beta}^j\varphi_j A_{V,j}\bar{E}_{j-1}^{\psi-1}E_j \tag{B.80}$$

$$E_{t>T}: \quad \tilde{\beta}^t\varphi_t A_{E,t} = \tilde{\beta}^t\mu_t A_{E,t} + \tilde{\beta}^t\varphi_t p_{E,t} + \psi\sum_{j=t+1}^{\infty}\tilde{\beta}^j\varphi_j A_{V,j}\bar{E}_{j-1}^{\psi-1}E_j \tag{B.81}$$

$$A_{E,t}: \quad \tilde{\beta}^t\varphi_t E_t + \tilde{\beta}^{t+1}\kappa_{t+1}^E(1+\eta_E R_{E,t+1}^{1-\lambda}) = \tilde{\beta}^t\mu_t E_t + \tilde{\beta}^t\kappa_t^E \tag{B.82}$$

$$A_{N,t}: \quad \tilde{\beta}^t\mu_t(1-\alpha)K_t^\alpha L_t^{1-\alpha}A_{N,t}^{-\alpha} + \tilde{\beta}^{t+1}\kappa_{t+1}^N(1+\eta_N R_{N,t+1}^{1-\lambda}) = \tilde{\beta}^t\kappa_t^N \tag{B.83}$$

$$R_{E,t}: \quad \xi_t = \kappa_t^E(1-\lambda)\eta_E R_{E,t}^{-\lambda}A_{E,t-1} \tag{B.84}$$

$$R_{N,t}: \quad \xi_t = \kappa_t^N(1-\lambda)\eta_N R_{N,t}^{-\lambda}A_{N,t-1}. \tag{B.85}$$

To start, I characterize the marginal costs and benefits of energy use, focusing on outcomes prior to period $T+1$. Rearranging (B.80) and dividing through by $\tilde{\beta}^t$ to focus on current values gives

$$\underbrace{(\varphi_t-\mu_t)A_{E,t}}_{\text{benefits of energy use}} = \underbrace{\varphi_t p_{E,t}}_{\text{private costs}} \underbrace{+\tilde{\beta}^{-t}\Omega + \psi\sum_{j=t+1}^{\infty}\tilde{\beta}^{j-t}\varphi_j A_{V,j}\bar{E}_{j-1}^{\psi-1}E_j}_{\text{external costs}}. \tag{B.86}$$

Note that φ_t gives the shadow value of increasing output, while μ_t is the shadow value of relaxing the Leontief production constraint. Along the optimal path, μ_t must also be the marginal cost of of relaxing the Leontief production constraint. Thus, we can interpret $(\varphi_t-\mu_t)$ as the net marginal benefit of increasing output in period t, after accounting for the Leontief constraint. The left-hand side (LHS) of (B.86) is therefore the marginal benefit of increasing energy use. The first term on the right-hand side (RHS) gives the value of the marginal extraction cost necessary to increase energy use. In a laissez-faire equilibrium, this would be the private cost of paid by energy suppliers . Also, $\tilde{\beta}^{-t}\Omega$ is the marginal cost of using up the energy budget in terms of period t utility, which implicitly captures the climate change externality. Noting that $\frac{dp_{E,\hat{t}}}{dE_t}|_{\hat{t}>t} = \psi A_{V,j}\bar{E}_{j-1}^{\psi-1}$, the last term on the RHS is the marginal impact of increasing E_t on future extraction costs, discounted back to period t. Together, the last two terms on the RHS are the marginal external costs that are ignored by firms in a laissez-faire equilibrium.

To compare the social planner and decentralized solutions, it will be helpful to scale everything to be relative to the value of the final good in period t (the numeraire). So, divide (B.86) through by $A_{E,t}\varphi_t$ and rearrange to obtain

$$1 - \frac{\mu_t}{\varphi_t} = \frac{p_{E,t} + \tilde{\Omega}_t + \tilde{\Upsilon}_t}{A_{E,t}}, \tag{B.87}$$

where $\tilde{\Omega}_t \equiv \varphi_t^{-1}\tilde{\beta}^{-t}\Omega$ and $\tilde{\Upsilon}_t \equiv \psi \sum_{j=t+1}^{\infty} \tilde{\beta}^{j-t}\frac{\varphi_j}{\varphi_t} A_{V,j}\bar{E}_{j-1}^{\psi-1}E_j$ are the marginal external costs relative to the value of the final good in period t.

Next, I turn to the savings decision. Dividing (B.79) through by $\tilde{\beta}_t$ yields

$$\varphi_t = \tilde{\beta}\mu_{t+1}\alpha K_{t+1}^{\alpha-1}(A_{N,t+1}L_{t+1})^{1-\alpha}. \tag{B.88}$$

When the social planner is considering how to use the marginal final good, they have two options. They could consume it, increasing utility by φ_t. Alternatively, they could use it to increase capital in period $t+1$ and relax the Leontief constraint by $\alpha K_{t+1}^{\alpha-1}(A_{N,t+1}L_{t+1})^{1-\alpha}$. The social value of relaxing the constraint is μ_{t+1}, and $\tilde{\beta}$ is the effective discount rate. To rewrite this intertemporal optimality condition in terms of consumption, divide through by φ_{t+1} and substitute in from (B.78) to arrive at

$$\frac{C_{t+1}}{C_t} = \frac{\mu_{t+1}}{\varphi_{t+1}}\alpha\tilde{\beta}K_{t+1}^{\alpha-1}(A_{N,t+1}L_{t+1})^{1-\alpha}. \tag{B.89}$$

This is a standard Euler equation for a growth model, except for the extra term $\frac{\mu_{t+1}}{\varphi_{t+1}}$, which gives the value of relaxing the Leontief constraint relative to the value of increasing total output. The two are different, because relaxing the Leontief constraint only contributes to utility after increasing energy use. To incorporate the costs of increasing energy use, plug in from (B.87) to get the Euler equation for the social planner

$$\frac{C_{t+1}}{C_t} = \left[1 - \frac{p_{E,t+1} + \tilde{\Omega}_{t+1} + \tilde{\Upsilon}_{t+1}}{A_{E,t+1}}\right]\alpha\tilde{\beta}K_{t+1}^{\alpha-1}(A_{N,t+1}L_{t+1})^{1-\alpha}. \tag{EE-SP}$$

Comparing the Euler equation in the decentralized equilibrium, (EE-DE), to the Euler equation for the social planner, (EE-SP), gives the following result.

Remark B.1. *To implement the optimal allocations, it is necessary for the government to set* $\tau_t^u = \tilde{\Omega}_t + \tilde{\Upsilon}_t,\ \forall t \geq 1$.

The intuition is straightforward. Energy use in period t has two impacts on future economic outcomes. First, it decreases the remaining energy budget that can be used while staying within the environmental target. The marginal cost of reducing the budget in captured by $\tilde{\Omega}_t$. Second, using energy in period t increases the extraction cost in future periods, which is captured by $\tilde{\Upsilon}_t$. In the absence of policy, neither of these costs are internalized by firms, which leads to overuse of energy. This can be corrected with a tax set equal to the un-internalized, intertemporal costs. This is standard Pigouvian reasoning, noting that using up the energy budget is essentially an externality when the environmental target is given exogenously.

It is somewhat unusual to identify this result through the Euler equation. Most existing work characterizes the optimal tax through the static first order conditions of the final good producer (e.g., Acemoglu et al., 2012; Hart, 2019). Given the Leontief production function, however, there is no optimality condition through which the tax rate affects the relative marginal product of energy and non-energy inputs. Conditional on the technology levels, energy taxes only affect energy use by influencing savings decisions and the scale of the economy. Since energy taxes work through savings decisions, there are a wide range of energy taxes in period 0 that yield identical patterns of consumption. I focus on the case where $\tau_0^u = \tilde{\Omega}_0 + \tilde{\Upsilon}_0$, which simplifies the characterization of the R&D allocations.

Now, I turn to evaluating R&D allocations. From the research arbitrage equation in the decentralized equilibrium, (RA-DE), it is clear that the tax will also affect the direction of R&D. The remaining policy question is whether it is necessary to also have η_t^u different from zero. In other words, is directed R&D policy necessary to implement the least-cost path, or is the energy tax sufficient? To characterize the marginal benefit of energy efficiency R&D, divide the first order condition for $A_{E,t}$, (B.82), through by $\tilde{\beta}^t$ and rearrange to arrive at

$$\underbrace{\kappa_t^E}_{\substack{\text{marginal}\\ \text{benefit}}} = \underbrace{(\varphi_t - \mu_t)\, E_t}_{\text{static benefit}} + \underbrace{\tilde{\beta}\kappa_{t+1}^E \left(1 + \eta_E R_{E,t+1}^{1-\lambda}\right)}_{\text{intertemporal benefit}}. \tag{B.90}$$

The LHS is the shadow value of increasing energy efficiency at time t and the RHS divides this into two terms. The first term captures the impact in the contemporary period. As discussed in the context of increasing energy use, the marginal benefit of increasing effective energy inputs is given by $(\varphi_t - \mu_t)$, which incorporates the costs necessary to increase potential output. In the decentralized model, $(\varphi_t - \mu_t)E_t$ is the private benefit of increasing $A_{E,t}$ to the final good producer.[49] The second expression on the RHS captures the intertemporal

[49]Capital good producers make R&D decisions. In principle, the incentives of the two types of firms

benefit of increasing energy efficiency. Specifically, $\frac{\partial A_{E,t+1}}{\partial A_{E,t}} = 1 + \eta_E R_{E,t}^{1-\lambda}$, and $\tilde{\beta}\kappa_{t+1}^E$ is the marginal value of increasing $A_{E,t+1}$, discounted back to period t. Firms maximize single period profits and do not internalize this intertemporal spillover. To derive the equivalent expression for non-energy technology, divide the first order condition for $A_{N,t}$, (B.83), by $\tilde{\beta}_t$ and rearrange to arrive at

$$\underbrace{\kappa_t^N}_{\substack{\text{marginal}\\\text{benefit}}} = \underbrace{\mu_t(1-\alpha)K_t^\alpha L_t^{1-\alpha} A_{N,t}^{-\alpha}}_{\text{static benefit}} + \underbrace{\tilde{\beta}\kappa_{t+1}^N \left(1 + \eta_N R_{N,t+1}^{1-\lambda}\right)}_{\text{intertemporal benefit}}. \tag{B.91}$$

Since there is a fixed quantity of R&D inputs, R&D policy can affect the direction of technical change, but not the overall level. Taking the ratio of the previous two equations gives

$$\frac{\kappa_t^E}{\kappa_t^N} = \frac{(\varphi_t - \mu_t)E_t + \tilde{\beta}\kappa_{t+1}^E(1 + \eta_E R_{E,t+1}^{1-\lambda})}{\mu_t(1-\alpha)K_t^\alpha L_t^{1-\alpha} A_{N,t}^{-\alpha} + \tilde{\beta}\kappa_{t+1}^N(1 + \eta_N R_{N,t+1}^{1-\lambda})}. \tag{B.92}$$

Along the optimal path, the relative marginal benefit of improving the two types of technology must be equal to the relative marginal costs. Rearranging the first order condition for $R_{E,t}$, (B.84), yields

$$\kappa_t^E = \underbrace{\xi_t}_{\substack{\text{Value of}\\\text{R\&D inputs}}} / \underbrace{\left((1-\lambda)\eta_E R_{E,t}^{-\lambda} A_{E,t-1}\right)}_{\text{R\&D Productivity}}. \tag{B.93}$$

Taking the ratio of first order conditions for $R_{E,t}$ and $R_{N,t}$, (B.84) and (B.85), and rearranging yields

$$\underbrace{\frac{\kappa_t^N}{\kappa_t^E}}_{\text{relative benefit}} = \underbrace{\frac{\eta_E R_{E,t}^{-\lambda} A_{E,t-1}}{\eta_N R_{N,t}^{-\lambda} A_{N,t-1}}}_{\text{relative cost}}. \tag{B.94}$$

The relative marginal cost of improving the technologies is the inverse of relative research productivity.

To compare the social planner allocation to the decentralized allocation, it will help to separate the static and intertemporal returns to improved technology. In the absence of policy, only the static benefits are internalized by firms. Multiplying both sides of (B.92) by

could be misaligned in the decentralized model, but this won't be the case with Leontief production. In a model with smooth substitution between energy and non-energy inputs, the capital good producer would have incentive to influence the final good producer's decision to substitute between energy and non-energy inputs.

$\frac{\mu_t(1-\alpha)K_t^\alpha L_t^{1-\alpha}A_{N,t}^{-\alpha}+\tilde{\beta}\kappa_{t+1}^N(1+\eta_N R_{N,t+1}^{1-\lambda})}{\mu_t(1-\alpha)K_t^\alpha L_t^{1-\alpha}A_{N,t}^{-\alpha}}$ gives

$$\left(1+\frac{\tilde{\beta}\kappa_{t+1}^N(1+\eta_N R_{N,t+1}^{1-\lambda})}{\mu_t(1-\alpha)K_t^\alpha L_t^{1-\alpha}A_{N,t}^{-\alpha}}\right)\cdot\frac{\kappa_t^E}{\kappa_t^N}=\frac{(\varphi_t-\mu_t)E_t}{\mu_t(1-\alpha)K_t^\alpha L_t^{1-\alpha}A_{N,t}^{-\alpha}}+\frac{\tilde{\beta}\kappa_{t+1}^E(1+\eta_E R_{E,t+1}^{1-\lambda})}{\mu_t(1-\alpha)K_t^\alpha L_t^{1-\alpha}A_{N,t}^{-\alpha}}. \tag{B.95}$$

Multiplying through by $\frac{\kappa_t^N}{\kappa_t^E}$ and rearranging gives

$$\left(1+\frac{\tilde{\beta}\kappa_{t+1}^N(1+\eta_N R_{N,t+1}^{1-\lambda})}{\mu_t(1-\alpha)K_t^\alpha L_t^{1-\alpha}A_{N,t}^{-\alpha}}\right)-\left(\frac{\tilde{\beta}\kappa_{t+1}^E(1+\eta_E R_{E,t+1}^{1-\lambda})}{\mu_t(1-\alpha)K_t^\alpha L_t^{1-\alpha}A_{N,t}^{-\alpha}}\right)\frac{\kappa_t^N}{\kappa_t^E}=\left(\frac{(\varphi_t-\mu_t)E_t}{\mu_t(1-\alpha)K_t^\alpha L_t^{1-\alpha}A_{N,t}^{-\alpha}}\right)\frac{\kappa_t^N}{\kappa_t^E}. \tag{B.96}$$

The first term on the RHS is now the relative static value of improving the two technologies, ignoring impacts on future research productivity. From (B.94), the second term gives relative research productivity. These are the two forces that influence capital good producers' R&D decisions in the absence of policy.

From (B.87),

$$\frac{\varphi_t-\mu_t}{\mu_t}=\frac{\frac{p_{E,t}+\tilde{\Omega}_t+\tilde{\Upsilon}_t}{A_{E,t}}}{1-\frac{p_{E,t}+\Omega_t+\Upsilon_t}{A_{E,t}}} \tag{B.97}$$

is the value of a marginal increase in effective energy inputs, relative to the value of a marginal increase in non-energy inputs. Also, the Leontief constraint implies that

$$\frac{E_t}{K_t^\alpha L_t^{1-\alpha}A_{N,t}^{-\alpha}}=\frac{A_{N,t}}{A_{E,t}}. \tag{B.98}$$

Plugging (B.94), (B.97), and (B.98) into (B.96) and taking a common denominator on the LHS gives

$$\begin{aligned}&\frac{\mu_t(1-\alpha)K_t^\alpha L_t^{1-\alpha}A_{N,t}^{-\alpha}+\tilde{\beta}\kappa_{t+1}^N(1+\eta_N R_{N,t+1}^{1-\lambda})-\frac{\kappa_t^N}{\kappa_t^E}\tilde{\beta}\kappa_{t+1}^E(1+\eta_E R_{E,t+1}^{1-\lambda})}{\mu_t(1-\alpha)K_t^\alpha L_t^{1-\alpha}A_{N,t}^{-\alpha}}\\&\qquad=(1-\alpha)^{-1}\left(\frac{\frac{p_{E,t}+\tilde{\Omega}_t+\tilde{\Upsilon}_t}{A_{E,t}}}{1-\frac{p_{E,t}+\tilde{\Omega}_t+\tilde{\Upsilon}_t}{A_{E,t}}}\right)\cdot\left(\frac{A_{N,t}}{A_{E,t}}\right)\cdot\left(\frac{\eta_E R_{E,t}^{-\lambda}A_{E,t-1}}{\eta_N R_{N,t}^{-\lambda}A_{N,t-1}}\right).\end{aligned} \tag{B.99}$$

Now, the RHS of the equation captures the relative benefit of improving each of the two technologies, when ignoring intertemporal spillovers. With $\tau_t^u=\tilde{\Omega}_t+\tilde{\Upsilon}_t$ – as in Remark B.1 – the RHS of this equation is identical to the research arbitrage equation in the decentralized equilibrium, (RA-DE).

To compare the social planner and decentralized solutions, it is necessary to rewrite the denominator of the LHS of (B.99) as the price of R&D inputs. Following the formulation in (PR-DE), I will focus on the value of improving non-energy technology. The full social value of R&D inputs relative to the final good is ξ_t/φ_t. In the decentralized solution, producers only internalize the static benefits of innovation. From (B.91), therefore, the final good producer will internalize a fraction $\mu_t(1-\alpha)K_t^{\alpha}L_t^{1-\alpha}A_{N,t}^{-\alpha}/\kappa_t^N$ of this total value. The capital good producer only receives a fraction α of this value, with the rest being paid to labor. Intuitively, multiplying these three expressions should yield the price of R&D inputs, when the tax corrects for all other externalities. To verify this, I use (B.85), multiply and divide by $A_{N,t}$, and use (B.87), respectively:

$$\begin{aligned}\frac{\alpha\xi_t}{\varphi_t\kappa_t^N}\left(\mu_t(1-\alpha)K_t^{\alpha}L_t^{1-\alpha}A_{N,t}^{-\alpha}\right) &= \left((1-\lambda)\eta_N R_{N,t}^{-\lambda}A_{N,t-1}\right)\cdot\left((1-\alpha)K_t^{\alpha}L_t^{1-\alpha}A_{N,t}^{-\alpha}\right)\cdot\left(\frac{\alpha\mu_t}{\varphi_t}\right)\\ &= (1-\lambda)(1-\alpha)\cdot\left(\frac{\eta_N R_N^{-\lambda}}{1+g_{A_N,t}}\right)\cdot\left(K_t^{\alpha}L_t^{1-\alpha}A_{N,t}^{1-\alpha}\right)\cdot\left(\frac{\alpha\mu_t}{\varphi_t}\right)\\ &= (1-\lambda)(1-\alpha)\cdot\left(\frac{\eta_N R_N^{-\lambda}}{1+g_{A_N,t}}\right)\cdot\left(\alpha\left(1-\frac{p_{E,t}+\tilde{\Omega}_t+\tilde{\Upsilon}_t}{A_{E,t}}\right)K_t^{\alpha}L_t^{1-\alpha}A_{N,t}^{1-\alpha}\right). \quad \text{(PR-SP)}\end{aligned}$$

Comparing price of research inputs in the decentralized equilibrium, (PR-DE), to the implicit price of research inputs for the social planner, (PR-SP), gives the following result.

Remark B.2. *Suppose that the government sets $\tau_t^u = \tilde{\Omega}_t+\tilde{\Upsilon}_t$ and $\tau_t^K = (1-\alpha)$ $\forall t$ in a decentralized equilibrium that implements the least-cost path. Then, $p_t^R = \frac{\alpha\xi_t}{\kappa_N^t\varphi_t}\left(\mu_t(1-\alpha)K_t^{\alpha}L_t^{1-\alpha}A_{N,t}^{-\alpha}\right)$ $\forall t$.*

Multiplying and dividing the LHS of (B.99) by $\frac{\alpha\xi_t}{\kappa_N^t\varphi_t}$ yields

$$\begin{aligned}&\frac{\left(\frac{\alpha\xi_t}{\kappa_N^t\varphi_t}\left(\mu_t(1-\alpha)K_t^{\alpha}L_t^{1-\alpha}A_{N,t}^{-\alpha}\right)\right)+\left(\frac{\alpha\xi_t}{\kappa_N^t\varphi_t}\right)\cdot\left(\tilde{\beta}\kappa_{t+1}^N(1+\eta_N R_{N,t+1}^{1-\lambda})-\frac{\kappa_t^N}{\kappa_t^E}\tilde{\beta}\kappa_{t+1}^E(1+\eta_E R_{E,t+1}^{1-\lambda})\right)}{\left(\frac{\alpha\xi_t}{\kappa_N^t\varphi_t}\left(\mu_t(1-\alpha)K_t^{\alpha}L_t^{1-\alpha}A_{N,t}^{-\alpha}\right)\right)}\\ &\qquad= (1-\alpha)^{-1}\left(\frac{\frac{p_{E,t}+\Omega_t+\Upsilon_t}{A_{E,t}}}{1-\frac{p_{E,t}+\Omega_t+\Upsilon_t}{A_{E,t}}}\right)\cdot\left(\frac{A_{N,t}}{A_{E,t}}\right)\cdot\left(\frac{\eta_E R_{E,t}^{-\lambda}A_{E,t-1}}{\eta_N R_{N,t}^{-\lambda}A_{N,t-1}}\right). \quad \text{(B.100)}\end{aligned}$$

Now, to see how the government can implement the least-cost path allocations in the decentralized equilibrium, apply the results from Remarks B.1 and B.2 and distribute $(\kappa_t^N)^{-1}$ to arrive at

$$\frac{p_t^R + \left(\frac{\alpha\xi_t}{\varphi_t}\right)\cdot\left(\tilde{\beta}\frac{\kappa_{t+1}^N}{\kappa_t^N}(1+\eta_N R_{N,t+1}^{1-\lambda}) - \tilde{\beta}\frac{\kappa_{t+1}^E}{\kappa_t^E}(1+\eta_E R_{E,t+1}^{1-\lambda})\right)}{p_t^R}$$
$$= (1-\alpha)^{-1}\left(\frac{\frac{p_{E,t}+\tau_t^u}{A_{E,t}}}{1-\frac{p_{E,t}+\tau_t^u}{A_{E,t}}}\right)\cdot\left(\frac{A_{N,t}}{A_{E,t}}\right)\cdot\left(\frac{\eta_E R_{E,t}^{-\lambda}A_{E,t-1}}{\eta_N R_{N,t}^{-\lambda}A_{N,t-1}}\right). \quad \text{(RA-SP)}$$

Comparing research arbitrage equation for the decentralized equilibrium, (RA-DE), to the research arbitrage equation for the social planner, (RA-SP), gives the following result.

Remark B.3. *Consider a decentralized equilibrium where* $\tau_t^u = \tilde{\Omega}_t + \tilde{\Upsilon}_t$ *and* $\tau^K = (1-\alpha)$ $\forall t$. *To implement the least-cost path, it is sufficient for the government to set the subsidy/tax on energy efficiency R&D according to*

$$\eta_t^u = \left(\frac{\alpha\tilde{\beta}\xi_t}{\varphi_t}\right)\cdot\left(\frac{\kappa_{t+1}^E}{\kappa_t^E}(1+\eta_E R_{E,t+1}^{1-\lambda}) - \frac{\kappa_{t+1}^N}{\kappa_t^N}(1+\eta_N R_{N,t+1}^{1-\lambda})\right)\ \forall t, \quad \text{(B.101)}$$

with $\eta_t^u > 0$ *being a subsidy and* $\eta_t^u < 0$ *being a tax.*

This is an intuitive result. From (B.90), $\frac{\kappa_{t+1}^E}{\kappa_t^E}(1+\eta_E R_{E,t+1}^{1-\lambda})$ is the value of intertemporal spillovers from improving $A_{E,t}$ relative to the overall value of improving $A_{E,t}$, with an identical interpretation for the expression involving κ_t^N. When all other externalities are corrected with separate instruments, R&D policy favors energy efficiency when the spillovers in that technology are a greater share of the total value of R&D in that technology. When the importance of spillovers is balanced between the two technologies, the optimal subsidy is zero, which captures the fact that subsidies can affect the direction of R&D, but not the overall quantity.

To further characterize the role of R&D policy, apply (B.84) and (B.85) to (B.101), yielding

$$\eta_t^u = \left(\frac{\beta\xi_{t+1}}{\varphi_t}\right)\cdot\left[\left(\frac{R_{E,t+1}^{\lambda}}{R_{E,t}^{\lambda}}\frac{(1+g_{A_E,t+1})}{(1+g_{A_E,t})}\right) - \left(\frac{R_{N,t+1}^{\lambda}}{R_{N,t}^{\lambda}}\frac{(1+g_{A_N,t+1})}{(1+g_{A_N,t})}\right)\right]. \quad \text{(B.102)}$$

The government subsidizes energy efficiency R&D in period t if and only if $g_{A_E,t+1} > g_{A_E,t}$ (equivalently, $g_{A_N,t+1} < g_{A_N,t}$) and taxes energy efficiency R&D if and only if $g_{A_E,t+1} < g_{A_E,t}$ (equivalently, $g_{A_N,t+1} > g_{A_N,t}$). It does neither on a BGP.

The results presented in this section are summarized in Proposition 3, which I reproduce below for convenience.

Proposition 3. *The government can implement the least-cost path using three policies: (1) a subsidy for capital good production* $\tau_t^K = (1-\alpha)\ \forall t$*, (2) a tax* ($\tau_t^u > 0\ \forall t$) *on energy use, and (3) a subsidy/tax* (η_t^u) *for energy efficiency R&D. In addition,* η_t^u *has the same sign as* $(g_{A_E,t+1} - g_{A_E,t})$*. In other words, the government subsidizes energy efficiency R&D in period* t *if energy efficiency grows faster in period* $t+1$ *than in period* t*, taxes energy efficiency R&D if the growth rate slows between* t *and* $t+1$*, and does neither if the growth rate is constant between periods.*

Proof. Follows from Remarks B.1 – B.3 and equation (B.102). □

Following the exact steps from this section, it is possible to derive (B.102) if $\psi = 0$ (i.e., exogenous energy prices) and $\bar{E}^{\text{target}} = \infty$. Combined with the rest of the results from this section, this yields Corollary 2 in the main text, which is reproduced below.

Corollary 2. *Consider the case where* $\psi = 0$ *(i.e., exogenous energy prices) and* $\bar{E}^{target} = \infty$*. The government can implement the least-cost path using two policies: (1) a subsidy for capital good production* $\tau_t^K = (1-\alpha)\ \forall t$ *and (2) a subsidy/tax* (η_t^u) *for energy efficiency R&D, where* η_t^u *has the same sign as* $(g_{A_E,t+1} - g_{A_E,t})$*.*

As stressed by Golosov et al. (2014), separate instruments correct separate market failures. Energy taxes address externalities from energy use, and R&D policy addresses intertemporal knowledge spillovers. The fact that the energy tax and R&D subsidy/tax correct separate externalities does not imply that R&D policy plays no role in implementing the least-cost path. As stressed throughout the paper, transition dynamics are important for meeting environmental targets. If technological growth rates vary along the transition path, then directed R&D policy will be necessary to implement to the LCP. There is no presumption, however, that R&D policy needs to favor energy efficiency.

Simulation Method. — After noting that the Leontief and R&D market clearing constraints will hold with equality, the objective function of the social planner can be written in terms of two choice variables per period: K_{t+1} and $R_{E,t}$. The cumulative energy constraint can also be written in terms of these two variables. To find the allocations that maximize this simplified objective function subject to the constraint, I use the *fmincon* numerical procedure in *Matlab*. I approximate the infinite horizon problem with a finite horizon problem of 1000 years (100 periods). In Section 5.1.2, I focus on outcomes over a 100-year (10 period) time span, which is the length of time that the environmental constraint is in effect. To calculate welfare, I focus on outcomes over 300 years (30 periods). The choice of the terminal date has no impact on these results.

B.9 The Cobb-Douglas Model

B.9.1 Overview

The standard approach in climate change economics is to treat energy as a Cobb-Douglas component of the aggregate production function (e.g., Nordhaus and Boyer, 2000; Golosov et al., 2014; Hassler et al., 2016; Barrage, 2020). The standard Cobb-Douglas production function is given by

$$Q_t^{CD} = K_t^{\gamma} E_t^{\nu} \left(B_t L_t\right)^{1-\gamma-\nu},$$

where B_t grows at an exogenous rate, g_B. Due to perfect competition,

$$\tau_t p_{E,t} = \nu K_t^{\gamma} E_t^{\nu-1} \left(B_t L_t\right)^{1-\gamma-\nu}, \tag{B.103}$$

where τ_t is again the value-added tax on energy. Aggregate energy use is given by

$$E_t = \nu^{\frac{1}{1-\nu}} \left(\tau_t p_{E,t}\right)^{\frac{-1}{1-\nu}} K_t^{\frac{\gamma}{1-\nu}} \left(B_t L_t\right)^{\frac{1-\gamma-\nu}{1-\nu}}. \tag{B.104}$$

This, in turn, yields

$$Q_t^{\text{CD}} = \nu^{\frac{\nu}{1-\nu}} \left(\tau_t p_{E,t}\right)^{\frac{-\nu}{1-\nu}} K_t^{\frac{\gamma}{1-\nu}} \left(B_t L_t\right)^{\frac{(1-\gamma-\nu)}{1-\nu}}, \tag{B.105}$$

$$Y_t^{\text{CD}} = \left(1 - \frac{\nu}{\tau}\right) Q_t. \tag{B.106}$$

The energy expenditure share is given by

$$\theta_{E,t}^{CD} = \frac{\nu}{1 - \frac{\nu}{\tau_t}}.$$

In the absence of changes in the energy tax, the energy expenditure share is constant. As shown in Figure 2, this is consistent with the long-run data from the United States, but not the short-run data. In the Cobb-Douglas model, a tax on energy use – no matter how large – immediately generates a decline in energy use that is sufficient to leave the expenditure share essentially unchanged.[50] In other words, the Cobb-Douglas model has rapid transition

[50] In response to new energy taxes, there is actually a slight *decrease* in the energy expenditure share,

dynamics that are inconsistent with the data.

B.9.2 Calibration

The utility and energy extraction cost parameters are the same in the DTC and Cobb-Douglas models. I calibrate the production function of the Cobb-Douglas model to a laissez-faire balanced growth path (LFBGP) such that the levels and growth rates of output, population, flow energy use, cumulative energy use, and energy prices are the same as those on the LFBGP of the DTC model. This implies that the growth rates, but not the levels, of consumption and the capital stock are the same in the two models.

The levels of output and population are set by normalization. To ensure that the starting level of E/Y and the energy expenditure share are the same both models, I set $\frac{\nu}{1-\nu} = \theta_E^*$, which is observed in data. On the LFBGP of the DTC model, macroeconomic aggregates grow at factor $(1+g_Y^*) = (1+g_{A_N}^*)(1+n)$. To ensure that macroeconomic aggregates grow at the same rate on the LFBGP of the Cobb-Douglas model, I set $(1+g_B) = \left(\left(1+g_{A_N}^*\right)(1+n)\right)^{\frac{1-\gamma}{1-\gamma-\nu}} (1+g_E^*)^{\frac{-\nu}{1-\gamma-\nu}} (1+n)^{-1}$. All of the growth rates on the right-hand side are observed in the data.

All that remains is to determine the initial levels B_0 and K_0 such that the economy starts in a steady state. It is straightforward to write the Cobb-Douglas model in intensive form where

$$z_t = \frac{Z_t}{(B_t L_t)^{\frac{1}{1-\gamma-\nu}} (\tau_t \cdot p_{E,t})^{\frac{-\nu}{1-\gamma-\nu}}}, \tag{B.107}$$

for $Z_t = Y_t, K_t, C_t$. The steady state of the intensive form shows that

$$\begin{aligned} k^* &\equiv \frac{K_t}{(B_t L_t)^{\frac{1}{1-\gamma-\nu}} (p_{E,t})^{\frac{-\nu}{1-\gamma-\nu}}} \\ &= \nu^{\frac{-\nu}{\gamma+\nu-1}} \left(\frac{r^*}{\gamma}\right)^{\frac{1-\nu}{\gamma+\nu-1}}, \end{aligned}$$

and that

$$r^* = (1+g_b)(1+g_P^*)^{\frac{-\nu}{1-\gamma-\nu}} / \beta.$$

Combining these results gives the BGP value of k^* in terms of data and known parameters. Plugging $K_0 = k^* \cdot (B_0 L_0)^{\frac{1}{1-\gamma-\nu}} (\tau_t \cdot p_{E,t})^{\frac{-\nu}{1-\gamma-\nu}}$ into the Cobb-Douglas production function

which is due to the tax rebate. This effect is quantitatively unimportant.

yields B_0 (noting that Y_0 and L_0 are set by normalization and E_0 is pinned down by the ν and the price of energy). Plugging B_0 back into the definition of k^* gives the value of K_0.

B.9.3 Least-Cost Path

The social planner problem for the Cobb-Douglas model can be written as

$$\max_{\{K_{t+1},E_t\}_{t=0}^{\infty}} L_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \ln \left(K_t^{\gamma} E_t^{\nu} \left(B_t L_t\right)^{1-\gamma-\nu} - K_{t+1} - A_{V,t}^{-1} \left(\bar{E}_{-1} + \sum_{\hat{t}=t+1}^{\infty} E_{\hat{t}} \right)^{\psi} E_t \right), \tag{B.108}$$

$$\text{subject to: } \bar{E}_{-1} + \sum_{\hat{t}=0}^{T} E_{\hat{t}} < \bar{E}^{\text{target}}, \tag{B.109}$$

which takes advantage of the budget constraint, $Q_t^{\text{CD}} = C_t + K_{t+1} + p_{E,t}E_t$, and the energy price equation, $p_{E,t} = A_{V,t}^{-1}\bar{E}_{t-1}^{\psi}$. Let $L_0\Omega$ be the multiplier on the environmental constraint. The first order conditions for $t < T+1$ are

$$K_{t+1}: \ \frac{C_{t+1}}{C_t} = \tilde{\beta}\gamma K_t^{\gamma-1} E_t^{\nu} \left(B_t L_t\right)^{1-\gamma-\nu}, \tag{B.110}$$

$$E_t: \ \nu K_t^{\gamma} E_t^{\nu-1} \left(B_t L_t\right)^{1-\gamma-\nu} = p_{E,t} + \underbrace{\psi \sum_{\hat{t}=t+1}^{\infty} \tilde{\beta}^{\hat{t}-t} \frac{C_t}{C_{\hat{t}}} A_{V,\hat{t}} \bar{E}_{\hat{t}-1}^{\psi-1} E_{\hat{t}}}_{\tilde{\Upsilon}_t} + \underbrace{\tilde{\beta}^{-t} C_t \Omega}_{\tilde{\Omega}_t}. \tag{B.111}$$

It is straightforward to verify that (B.110) is the Euler equation for the decentralized model. Unlike the DTC case, the social planner in the Cobb-Douglas model does not need to adjust intertemporal decision-making in response to externalities from energy use. Comparing (B.111) and (B.103) shows that the least-cost path can be implemented with $\tau_t^u = \tilde{\Upsilon}_t + \tilde{\Omega}_t$, when using a per-unit tax instead of a value-added tax ($\tau_t^u + p_{E,t} = \tau_t p_{E,t}$). With smooth substitution between energy and non-energy inputs, the optimal tax adjusts intra-period allocations.

Simulation. — As explained above, the social planner problem in the Cobb-Douglas model can be written in terms of two choice variables, K_{t+1} and E_t. The cumulative energy constraint is specified in terms of E_t. To find the allocations that maximize the objective function subject to the constraint, I use the *fmincon* numerical procedure in *Matlab*. As in the numerical simulation of the least-cost path for the DTC model, I approximate the infinite horizon problem with a finite horizon problem of 1000 years (100 periods). In addition, the solution to the Cobb-Douglas model uses a 'backstop technology' that prevents any further increase in energy prices beyond some specified point. In sensitivity analyses for the work of Golosov et al. (2014), Barrage (2013) employs a similar strategy. The backstop technology appears after 550 years (55 periods). In Section 5.1.2, I focus on outcomes over a 100-year (10 period) time span, which is the length of time that the environmental constraint is in effect. To calculate welfare, I focus on outcomes over 300 years (30 periods). The choices of the terminal date and backstop date have no impact on dynamics over these earlier periods.

C Further Discussion of Data

C.1 Extraction Costs

Data. Extraction cost and energy availability estimates for coal, oil, and natural gas are taken from McGlade and Ekins (2015b).[51] Estimates of energy availability include known energy reserves in or scheduled to be in production, known resources not currently in production, estimates of reserve growth within known sources of energy, and estimates of undiscovered resources. Quantities correspond to *remaining ultimately recoverable resources* (RURR), the amount of energy that could be profitably extracted from the environment at some point in the future, even if it is not currently profitable to do so. This definition requires assumptions about future energy prices and technology. 'Additional occurrences' of coal, oil, and natural gas that fall outside the definition RURR are likely to be quite large (Rogner et al., 2012). I use the McGlade and Ekins (2015b) data to estimate the shape of the extraction cost curve, but not to estimate a limit to total energy availability.

Extraction costs in McGlade and Ekins (2015b) are estimated for current technology and include operating expenses, capital expenditure, and capital costs necessary to bring primary sources of energy to the market. They include exploration costs and exclude taxes. Costs were estimated separately for sub-categories of energy within the three broad types of fossil fuel.

Both cost and energy availability estimates are uncertain. McGlade and Ekins (2015b) construct a range of extraction cost curves at the country level for each fossil fuel. The publicly available global data are the median values from a Monte Carlo procedure that aggregates these country-level estimates. I combine their estimates to construct a cost curve for an aggregate fossil fuel energy composite.

[51] Data available at: `https://www.nature.com/articles/nature14016` (see source data for table 1). See McGlade (2014) for further details.

Aggregation. Energy availability is measured in different units for each broad type of energy. Oil is measured in barrels, natural gas is measured in cubic meters, and coal is measured by energy content. I use data from Rogner et al. (2012) to estimate the energy content of oil and natural gas.[52] Energy conversion factors are presented in Table B2. Figure C.6a plots the primary energy supply curves for each type of fossil fuel. Once availability is measured in terms of energy content for all sources, the data can be aggregated to derive a single primary energy cost curve, which is shown in Figure C.6b.

Different types of energy are converted from primary to final-use energy at different rates. To measure primary-to-final energy conversion factors, I take data on primary and final-energy use in the United States from the International Energy Agency (IEA). These are the same data used in Figure 1a, except that I now focus on the E_p/E_f ratio for individual types of fossil fuel energy, rather that country-level aggregates. Details on these calculations are provided below.

- **Oil.** I take total final-use energy consumption from oil products and subtract net imports of final-use oil products. Then, I divide this difference by the supply of primary energy from crude oil, natural gas liquids and refinery feedstocks. Data are averaged over the period 1971-2016.

- **Natural Gas.** Using the IEA data, I break natural gas usage into two categories: gas consumed directly as a final-use energy source and gas used to generate electricity. Data on natural gas usage is available from 1971-2016. By definition, the transformation efficiency for gas consumed as final-use energy is one. For electricity, the average transformation efficiency is taken from the EIA.[53] Due to data limitations on the heat rate of electricity production, the average efficiency is calculated using data from 2001-2016.

- **Coal.** The calculation for the transformation efficiency of coal is identical to the calculation for natural gas.

Figure C.6c plots the final-use energy supply curves separately for each source of energy, and Figure C.6d plots the aggregated curve.

[52] See table 7.3 in Rogner et al. (2012).

[53] Transformation efficiency is calculated as the heat rate of electricity generation divided by 3,412 Btu, which is the heat content of a kWh of electricity (https://www.eia.gov/tools/faqs/faq.php?id=107&t=3). Heat rate data are taken from Table A6, 'Approximate heat rates for electricity, and heat content of electricity,' of the Annual Energy Review (https://www.eia.gov/totalenergy/data/annual/).

Primary Type	Original Unit	Unit Conversion	Final/Primary
Oil	Barrels	5.71 GJ/bbl	85%
Nat. Gas	Cubic meters	0.04 GJ/m^3	73%
Coal	Joules	1 GJ/GJ	39%

Table B2: Energy Conversion. Unit conversion factors for primary energy taken from Rogner et al. (2012). 'Primary/final' is the efficiency of transforming primary sources of energy (e.g., coal, oil) into final sources of energy (e.g., electricity, gasoline). Efficiency data are taken from the IEA and EIA.

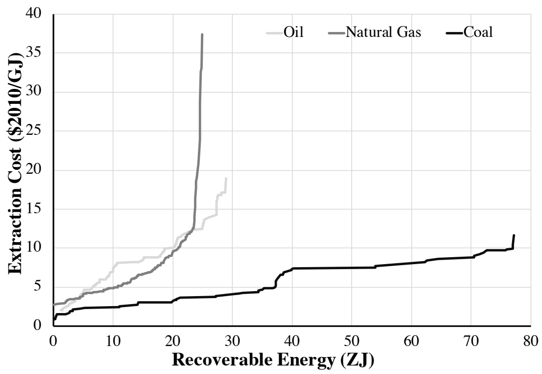
C.2 Energy Target

To construct a target for cumulative energy use that is consistent with the Paris Agreement, I use the results of an engineering study by Williams et al. (2014). They investigate the technical feasibility of achieving a target for flow CO_2 emissions. Specifically, they examine scenarios where carbon emissions in 2050 are 80 percent lower than 1990 levels, which is the United States' target for the Paris Agreement (Heal, 2017). Since climate change is a function of the stock of carbon in the atmosphere, I translate their findings into a target for cumulative energy use. This is consistent with the broader goal of the Paris Agreement to keep global average temperature change below 2°C above preindustrial levels (Dietz et al., 2018).

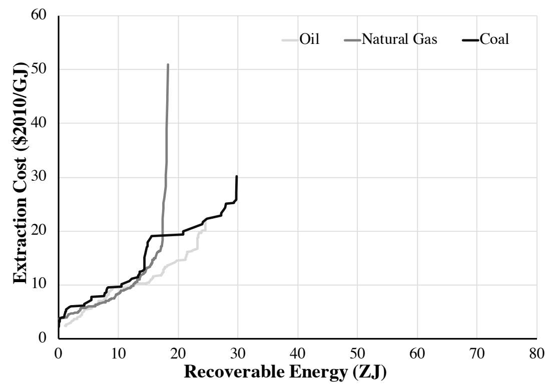
Williams et al. (2014) provide a path of final-use energy consumption from 2015-2050 that achieves the Paris target (Appendix Figure 1 on page A-6). They provide multiple scenarios, but energy use is very similar in all of them. I take the path of energy use from the baseline ('mixed') case. Their analysis indicates that energy use must decrease through 2050. To extend the estimate of cumulative energy use through 2114, I assume that flow energy use can reverse this decline starting 2051, while still staying within environmental limits. In particular, energy use is flat from 2051-2060, grows at half the laissez-faire BGP rate from 2061-2070, and then grows at the laissez-faire BFP rate until 2114.

There are two conservative assumptions in this analysis that potentially underestimate the importance of reduced final-use energy consumption for achieving the broader Paris Agreement goals. First, I assume that flow energy use abruptly reverse trend in 2051, the first year without information from Williams et al. (2014). Second, by specifying a date that the target disappears, I am assuming that all energy comes from clean primary sources by 2115. Most analyses, however, suggest that carbon capture and storage and/or CO_2 removal are necessary to achieve net zero emissions within the next 100 years (Fuss et al., 2014; Gasser et al., 2015). Energy use is still socially costly if these strategies are necessary.

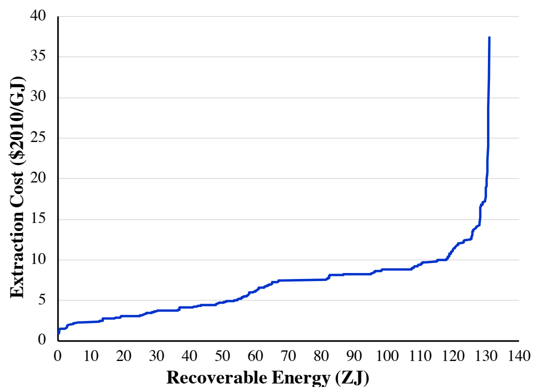
Figure C.7 shows the resulting path of energy use that defines the cumulative target. The



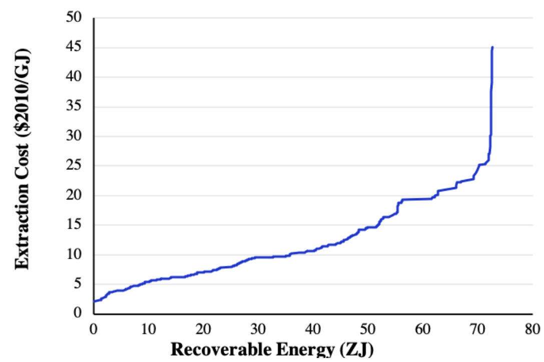
(a) Primary Energy



(b) Final-Use Energy



(c) Primary Energy



(d) Final-Use Energy

Figure C.6: Extraction Cost Curves. Estimates are originally from McGlade and Ekins (2015b). Conversion factors are given in Table B2.

area under the solid curve is the target for cumulative energy use. For ease of comparison, I also show the path where flow energy use grows at a rate consistent with the laissez-faire BGP. To translate the target to a unit-less measure, I normalize by flow energy use in 2005. The model is solved in ten year periods. For ease of comparison, I approximate the area under the curve by adding up the height at ten year intervals. This yields $E^{\text{target}} = 9.3 \cdot E_{2005}$. Along the laissez-faire BGP, cumulative energy use over the period 2015-2114 is 14.4 times E_{2005}.

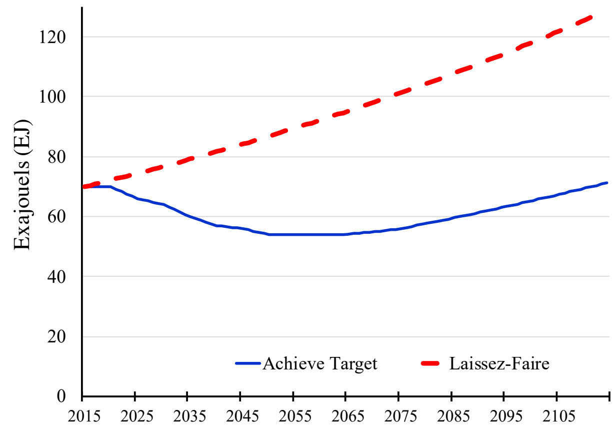
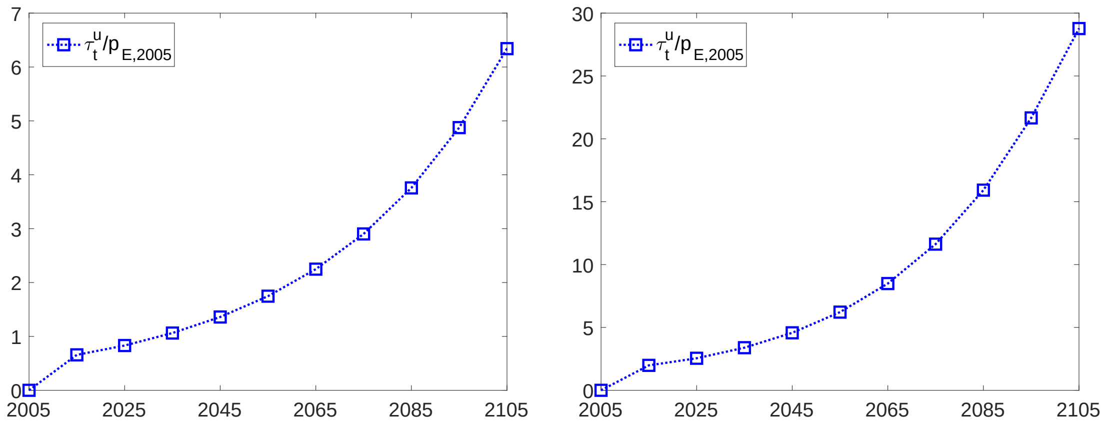


Figure C.7: Building the Energy Target

D Additional Quantitative Results

D.1 Comparing Taxes in the DTC and Cobb-Douglas Models

A key message of Section 5.1.1 is that energy use responds more slowly to environmental policy in the DTC model, when compared to the Cobb-Douglas model. To make this point, Section 5.1.1 compares how cumulative energy use in each of the two models reacts to an identical path of taxes. An alternate way to study the difference between the models is to study the difference in taxes needed to achieve the same cumulative energy use in the two models. To this end, Appendix Figure D.8 plots $\tau_t^u/p_{E,2005}$ along the least-cost path in both models. Panel (a) is a recreation of Figure 7a from the main text. Panel (b) differs from Figure 8a, because it normalizes taxes by $p_{E,2005}$ instead of $p_{E,t}$, and it does not include the path of R&D policy. The paths of taxes in the two models have a similar shape, but the magnitude is much greater in the DTC model. In 2015, the per-unit tax is almost five times larger in the DTC model. The fact that energy use reacts slowly to policy in the DTC model means that much more aggressive policy is necessary to achieve the environmental policy target. Given the difference in the path of energy use over time, the difference in taxes is not the same as the difference in tax-inclusive prices.



(a) Cobb-Douglas Model

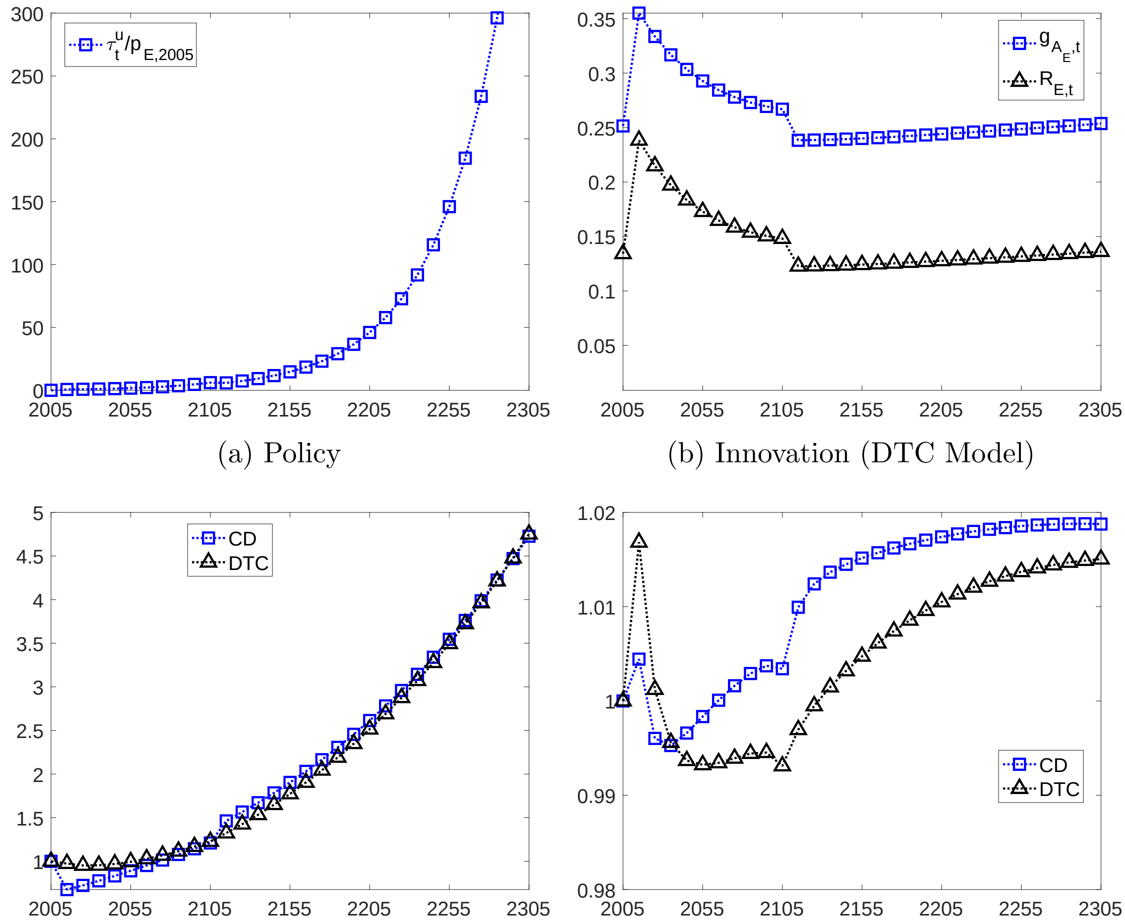
(b) DTC Model

Figure D.8: Tax Comparison

D.2 Dynamics through 2305

Figure D.9 extends the results in Figure 7 through 2305. Panel (a) shows the per-unit energy tax relative to the price of energy in the initial period ($\tau_t^u/p_{E,2005}$). The environmental target is in effect through 2105. There is a downward adjustment in energy taxes in 2115, but they continue to grow afterwards. The government continues to tax energy to correct for the extraction cost externality. Energy taxes grow slightly faster than extraction costs after 2105. The downward adjustment in energy taxes in 2115 reduces the incentive for energy efficiency R&D in the DTC model (panel b). Energy use in the Cobb-Douglas model reacts more quickly to the changes in the tax rate (panel c). In the long run, energy use is virtually identical in the two models. Consumption increases in both models once the target is no longer in effect and remains permanently above LFBGP levels, reflecting the welfare benefits of correcting the extraction cost externality (panel d).

Figure D.10 extends the results in Figure 8 through 2305. Panel (a) shows the per-unit energy tax relative to the contemporaneous price of energy ($\tau_t^u/p_{E,t}$). As in the Cobb-Douglas model, the energy tax shifts downward in 2115, the first period after the environmental target is no longer in effect. The ratio $\tau_t^u/p_{E,t}$ drops below one and only increases slightly over the next 200 years. The drop in taxes also leads to a fall in energy efficiency R&D (panel b). After the drop, the low level of energy efficiency R&D leads to an increase in the energy expenditure share, and energy efficiency R&D begins to increase again. In the long run, R&D allocations approximately converge back to their LFBGP levels. Since the growth rate of energy efficiency is increasing after 2105, policy promotes energy efficiency R&D (panel



(a) Policy

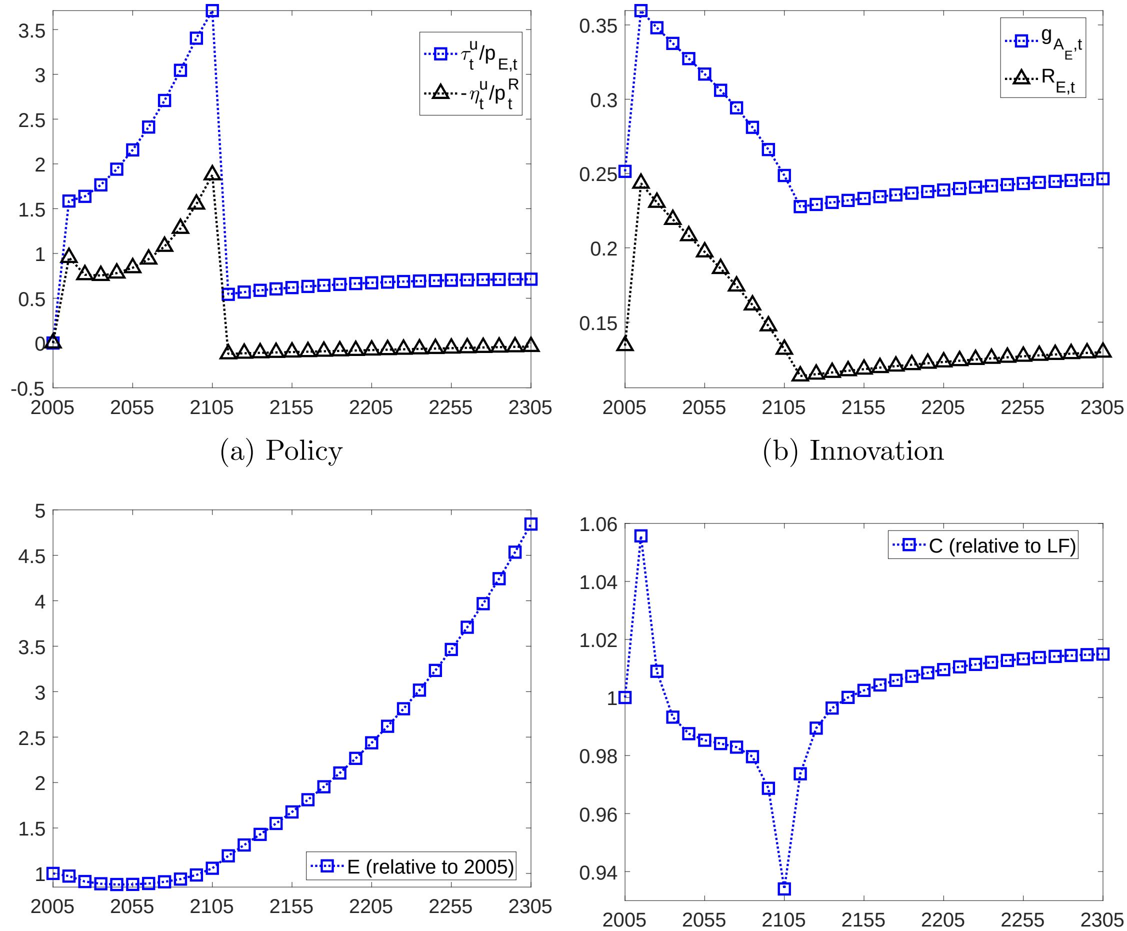
(b) Innovation (DTC Model)

(c) Energy (relative to 2005)

(d) Consumption

Figure D.9: Cobb-Douglas Comparison through 2305

a). Energy use grows rapidly after 2105 and is similar to energy use in the Cobb-Douglas model (panel c). Consumption also increases after 2105 and reaches a path that lies about 2 percent above the LFBGP level, reflecting the benefit of using policy to counteract the extraction cost and knowledge spillover externalities (panel d).



(a) Policy (b) Innovation

(c) Energy

Figure D.10: Least-Cost Path through 2305

D.3 Robustness for Cobb-Douglas Comparison

Figure D.11 recreates Figure 7 with less curvature in the R&D function, specifically $\lambda = 0.21$. This value, taken from Fried (2018), is significantly less than the baseline estimate of $\lambda = 0.40$, which is in turn less than the common finding of quadratic research costs ($\lambda = 0.50$) in quantitative models of endogenous growth (e.g., Acemoglu et al., 2018; Akcigit and Kerr, 2018). This alternative calibration does not affect the Cobb-Douglas model. The qualitative pattern of the results in the DTC model is unchanged. Since diminishing returns in the R&D function are less important, firms reallocate slightly more R&D inputs towards energy efficiency in response to the tax (panel b). In other words, the DTC model is more flexible than in the baseline calibration. As a result, the same path of taxes leads to a greater decrease in energy use (panel c). With the faster adjustment, the DTC models misses the environmental target by 6 percent. The path of consumption and lifetime utility of the representative household are similar to the baseline case. The impact of the taxes on the lifetime utility of the representative household is equivalent to increasing LFBGP consumption by 0.3 percent. Thus, all of the key results hold with this lower value of $\lambda = 0.21$.

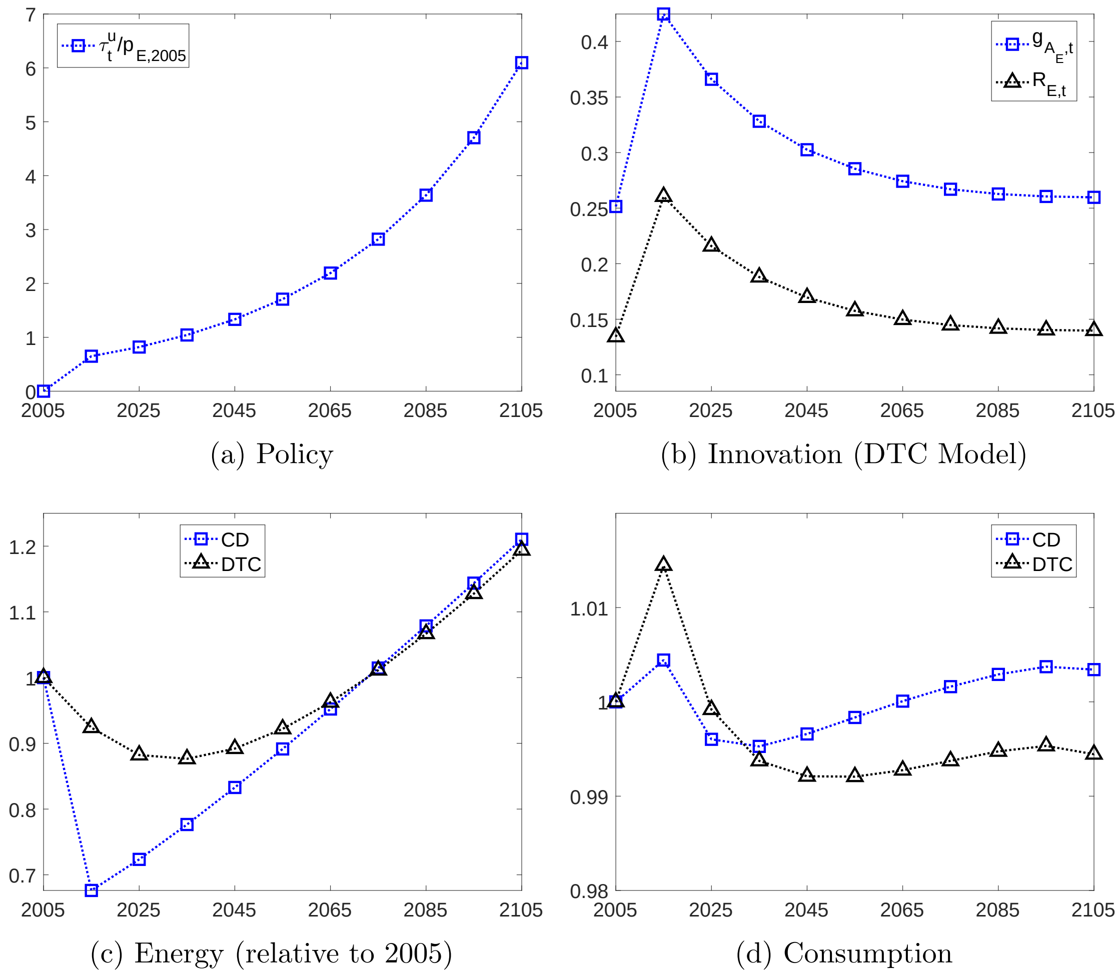
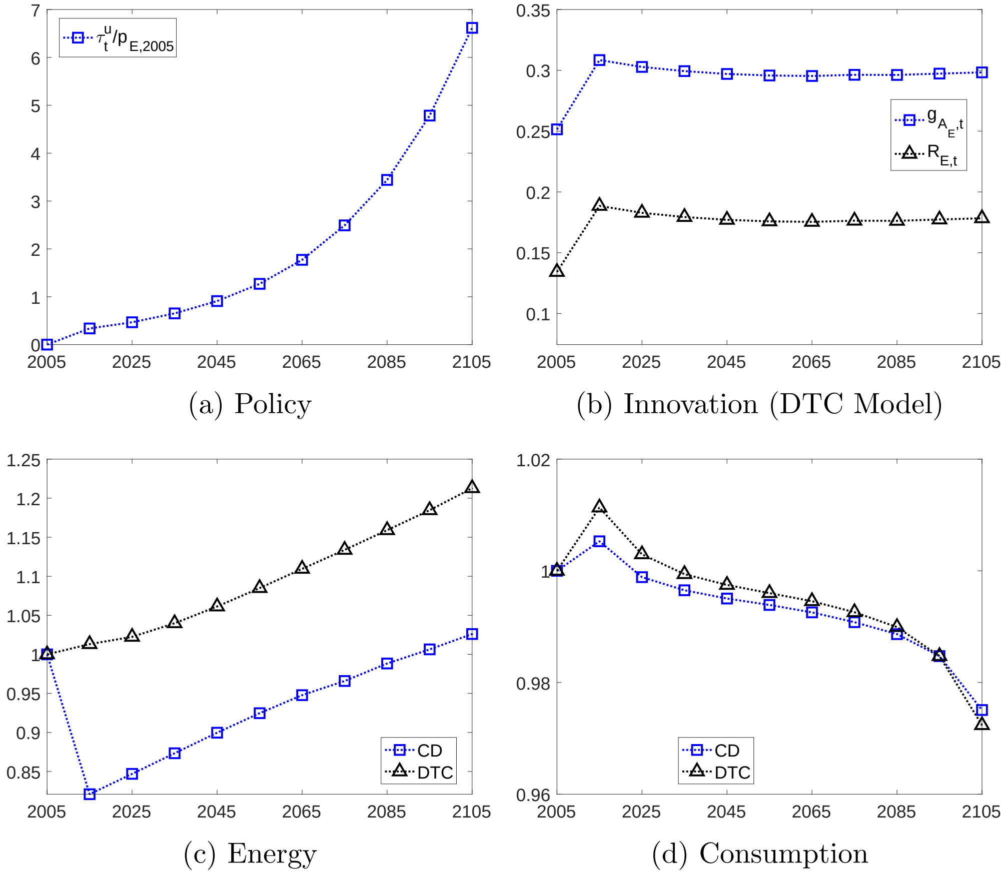


Figure D.11: Cobb-Douglas Comparison with $\lambda = 0.21$

Figure D.12 recreates Figure 7 for the case where energy prices grow exogenously at the LFBGP rate. In other words, it repeats the baseline calibration with $\psi = 0$. Changes to the calibration of energy supply affect both the Cobb-Douglas and DTC models. With exogenous energy prices, there is no extraction cost externality and tax-inclusive energy prices are the same in both models. Since energy prices do not respond to cumulative extraction, the Cobb-Douglas models hits the target with lower taxes (panel a). The lower



(a) Policy (b) Innovation (DTC Model)

(c) Energy (d) Consumption

Figure D.12: Cobb-Douglas Comparison with Exogenous Energy Prices ($\psi = 0$)

taxes lead to less reallocation of R&D inputs in the DTC model (panel b). The gap in energy use is larger and does not disappear over time (panel c). The DTC model misses the target by 19 percent. The paths of consumption in the two models are similar. Consumption in the DTC model is higher for most of the first 100 years. The impact of the taxes on the lifetime utility of the representative household is equivalent to a 0.7 percent decrease in LFBGP consumption in the DTC model and a 0.4 percent decrease in the Cobb-Douglas model. Unlike the baseline case, the utility of the representative household decreases in the Cobb-Douglas model, because there is no extraction cost externality. Overall, these results demonstrate that none of the main results are driven by the assumption of endogenous energy prices.

Figure D.13 recreates Figure 7 under the assumption that technological progress in the energy extraction sector occurs at the same speed as technological progress in the rest of the economy ($g_{A_V} = 0$). In this case, all of the observed increase in energy prices must be driven by the convexity in the extraction cost curve. In this alternative calibration, $\psi = 3.64$, compared to $\psi = 1.26$ in the baseline calibration. A high value of ψ implies that the extraction cost externality is large. Indeed, in this scenario, the energy target is no longer binding in the Cobb-Douglas model. Cumulative energy use from 2015 to 2105 is a little over 3 percent below the target (approximately 89 times 2005 use). The path of energy use is depicted in panel (c). Given the larger benefits from reduced energy use, the taxes that implement the least-cost path are significantly higher than in the baseline case (panel a). In the DTC model, these higher taxes lead to a faster reallocation of R&D inputs (panel b) and reductions in energy use (panel c), when compared to the baseline calibration. Cumulative energy use in the DTC model is still 4 percent above the environmental target. Unlike the baseline results, flow energy use is higher in the Cobb-Douglas model starting in 2075. The large reduction in Cobb-Douglas energy use early in the century leads to significantly lower energy prices, which in turn spurs economic growth and leads to rapid growth in both energy use and consumption (panels c and d). Since energy use reacts more slowly in the DTC model, it experiences neither the sharp initial drop in energy, nor the consequences of lower energy prices. In the Cobb-Douglas model, the impact of the taxes on the lifetime utility of the representative household is equivalent to a 2.6 percent increase in LFBGP consumption, compared to a 1.9 percent increase in the DTC model. Overall, the results demonstrate that the findings presented in the main text continue to hold with the significantly higher value of $\psi = 3.64$.

Figure D.14 recreates Figure 7 under the assumption that energy prices are exogenous and constant. This is a scenario where technological progress in the energy extraction sector always endogenously adjusts to offset any potential increase in energy prices driven by the convexity in the extraction cost curve. The primary goal of this analysis is to demonstrate that the difference in energy use between the two models does not depend on the assumption of increasing energy prices in the absence of policy. To perform the analysis, I use the baseline calibration, but assume that energy prices are constant from 2005 onward.[54] This captures the case where the data used for the calibration describe a time period with rising energy prices that was a deviation from the true long-run equilibrium with constant prices. The key difference with the main analyses is that the two models are not on a BGP in the absence of policy. As a result, they do not have the same growth rates of energy use or the other macroeconomic aggregates. Since the Cobb-Douglas model reacts more quickly to changes in tax-inclusive energy prices, constant energy prices lead to faster growth of energy use in

[54] I assume that prices begin to rise again in 2315. This has no effect on the dynamics I study, but simplifies the computational solution to the DTC model.

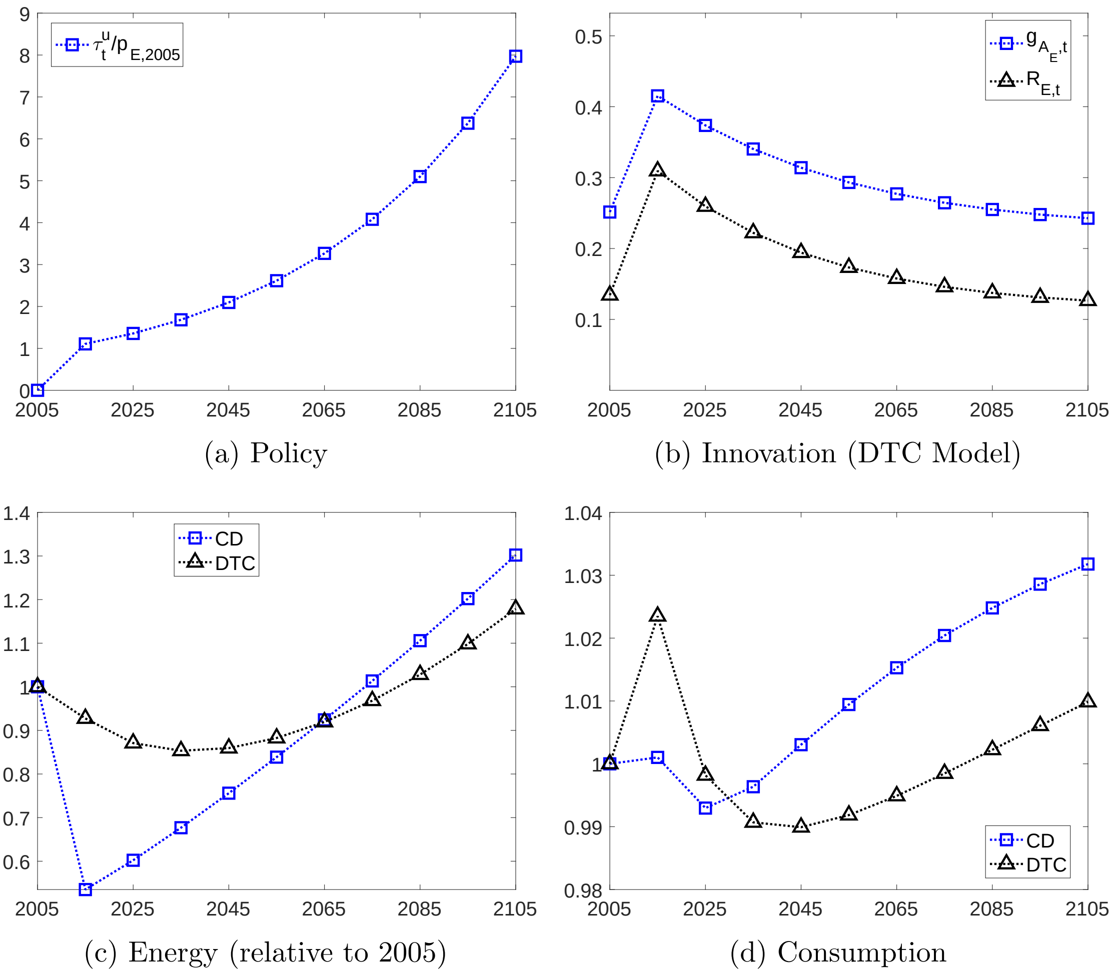


Figure D.13: Cobb-Douglas Comparison with $\psi = 3.64$

the Cobb-Douglas model in the absence of policy.

Since energy prices are lower than in the baseline case, the tax rate necessary to achieve the environmental target is higher (panel a). This leads to a significant reallocation of R&D inputs in the DTC model (panel b). Energy use responds more slowly in the DTC model, which misses the cumulative energy use target by 18 percent. This occurs even though energy use is higher in the Cobb-Douglas model in the absence of policy. Since there are no extraction cost externalities, the high tax rates lead to large decreases in consumption in both models (panel d). As in the earlier results with exogenous prices (Figure D.12), the consumption loss is larger in the Cobb-Douglas model, but the difference now increases over time. However, this is an imperfect comparison, because the growth rate of consumption is higher in the Cobb-Douglas model in the absence of policy. Overall, the results demonstrate

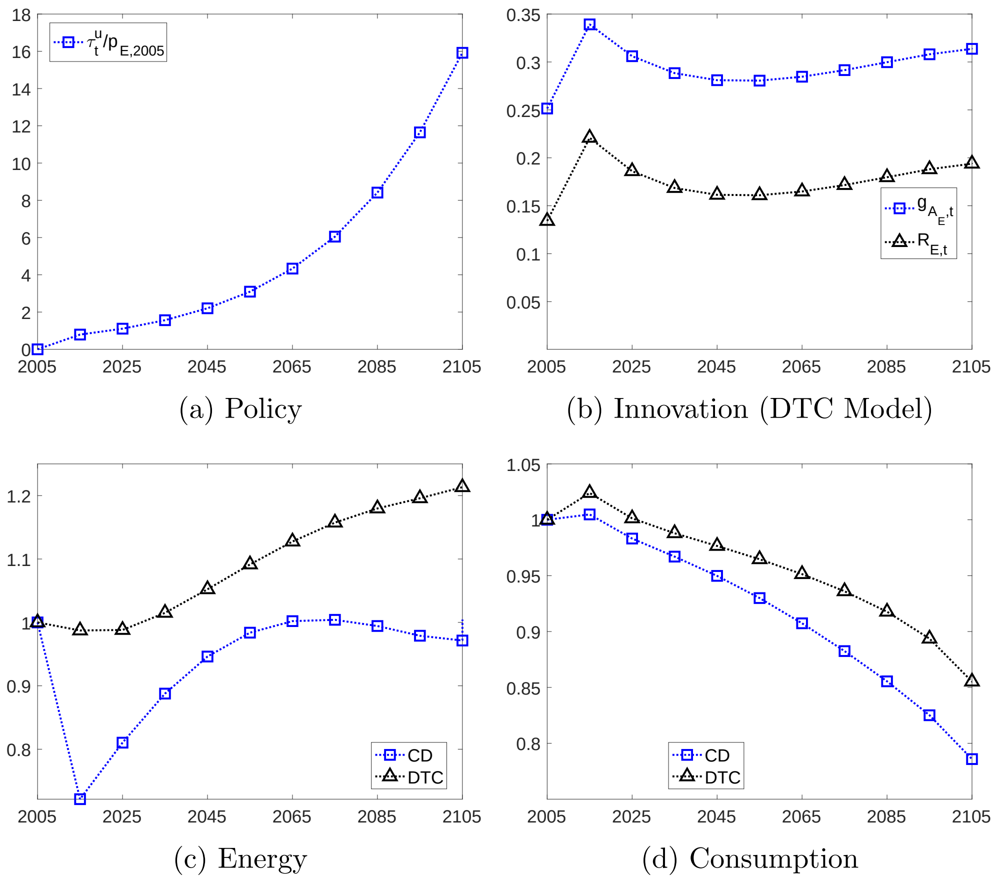
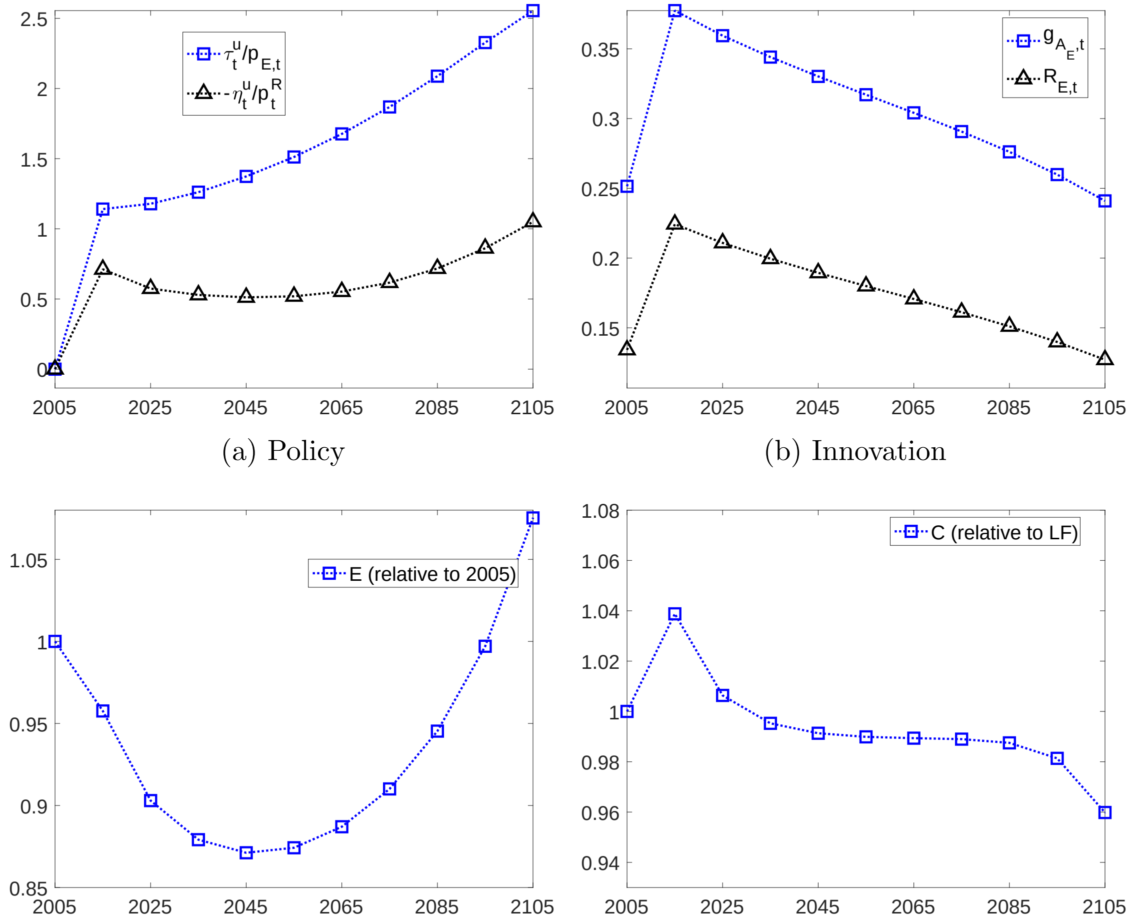


Figure D.14: Cobb-Douglas Comparison with Constant and Exogenous Energy Prices $(g_{A_V}, \psi = 0)$

that the difference in energy use between the two models is not driven by the assumption of increasing energy prices in the absence of energy taxation.

D.4 Robustness for Least-Cost Path in DTC Model

Figure D.15 recreates Figure 8 with less curvature in the R&D function, specifically $\lambda = 0.21$. This value, taken from Fried (2018), is significantly less than the baseline estimate of $\lambda = 0.40$, which is in turn less than the common finding of quadratic research costs ($\lambda = 0.50$) in quantitative models of endogenous growth (e.g., Acemoglu et al., 2018; Akcigit and Kerr, 2018). The qualitative pattern of the results is unchanged. Since diminishing returns in the R&D function are less important, smaller changes in $R_{E,t}$ are necessary to achieve the environmental target (panel b). This is also reflected in the fact that the energy taxes and R&D taxes are slightly smaller in magnitude than in the baseline case (panel a). The path



(a) Policy

(b) Innovation

(c) Energy

(d) Consumption

Figure D.15: Least-Cost Path with $\lambda = 0.21$

of energy is virtually identical to Figure 8 (panel c). The path of consumption has a similar shape with less dramatic swings, which reflects the smaller policy interventions (panel d). The impact of implementing the least-cost path on the lifetime utility of the representative household is equivalent to a 0.2 percent increase in LFBGP consumption. The change in lifetime utility is now positive, but remains smaller than the gain in the Cobb-Douglas model. Thus, all of the key results hold with this lower value of $\lambda = 0.21$.

Figure D.16 recreates Figure 8 with exogenous energy prices ($\psi = 0$). In this case, all of the observed increase in energy prices is driven by slow technological progress in the energy extraction sector. Since energy prices do not react to policy interventions, lower energy taxes and subsidies are necessary to implement the least-cost path (panel a). The paths of innovation and energy use are virtually identical to the baseline findings (panels b and

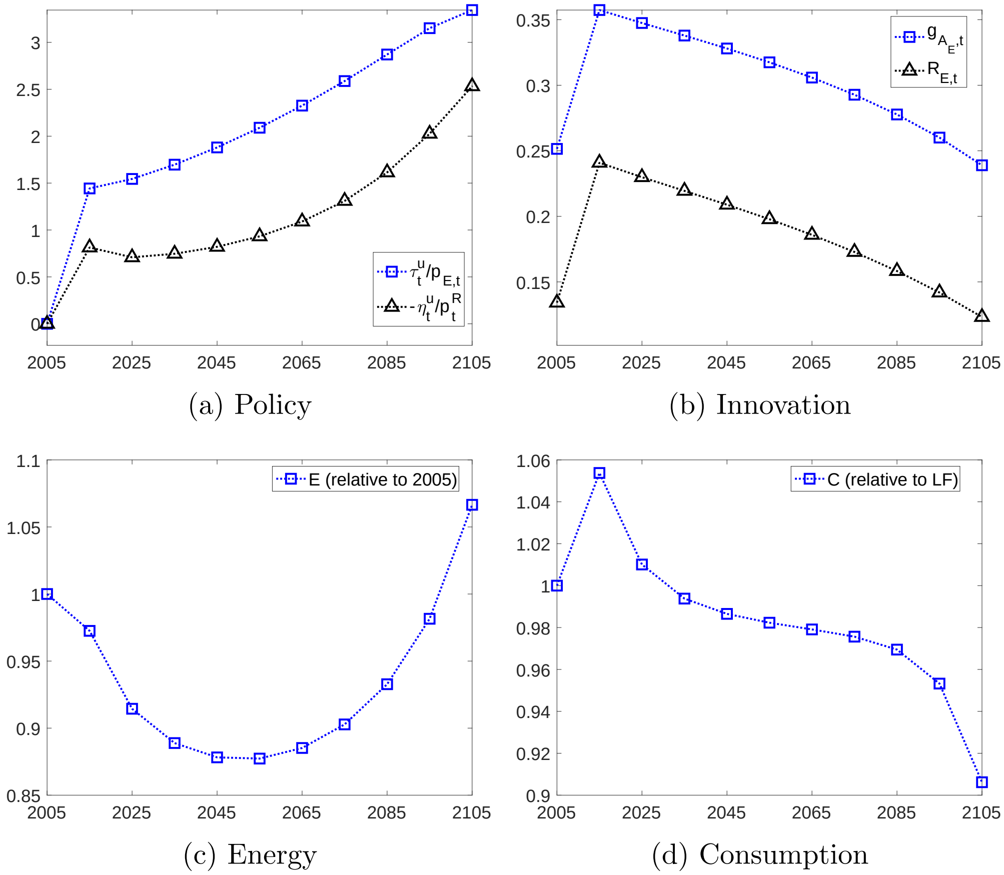
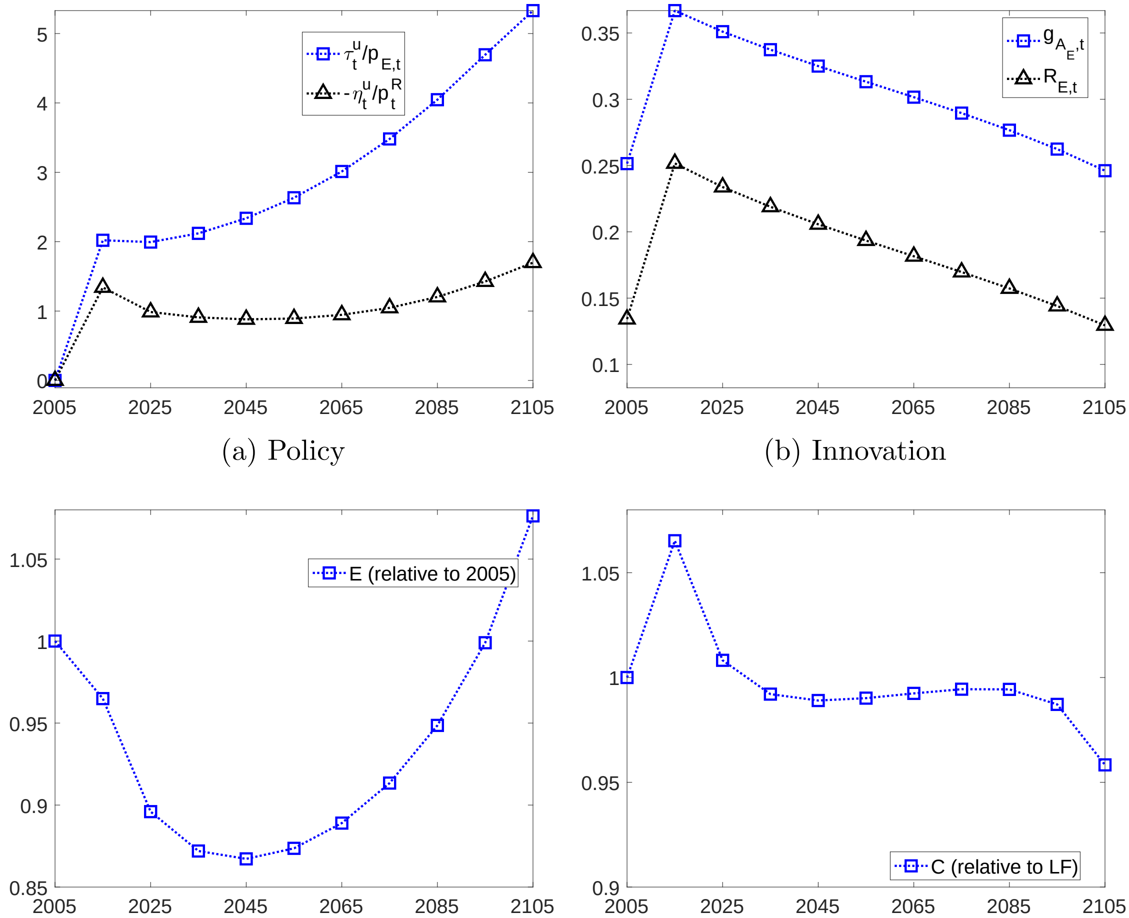


Figure D.16: Least-Cost Path with Exogenous Energy Prices ($\psi = 0$)

c). Since there are no extraction cost externalities, consumption and welfare are lower than in the baseline case (panel d). The overall impact of implementing the least-cost path on the lifetime utility of the representative agent is equivalent to an 1.6 percent decrease in LFBGP consumption. As in the baseline case, the loss in lifetime utility is greater in the DTC model than in the Cobb-Douglas model. Overall, the results demonstrate that the findings presented in the main text do not depend on the assumption of endogenous energy prices.

Figure D.17 recreates Figure 7 under the assumption that technological progress in the energy extraction sector occurs at the same speed as technological progress in the rest of the economy ($g_{A_V} = 0$). In this case, all of the observed increase in energy prices must be driven by the convexity in the extraction cost curve. In this alternative calibration, $\psi = 3.64$, compared to $\psi = 1.26$ in the main results. With a more convex extraction cost curve, energy prices react more strongly to changes in energy use. To achieve the environmental target,

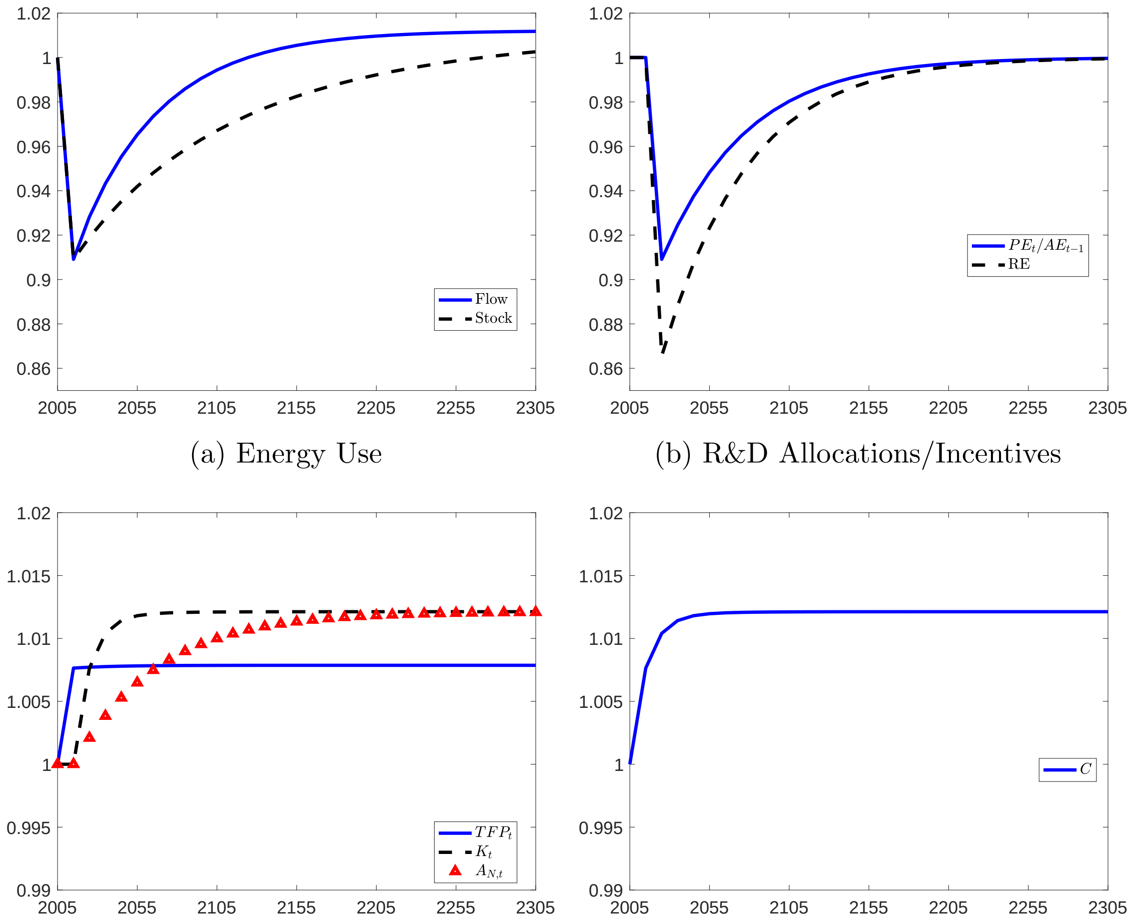


(a) Policy (b) Innovation

(c) Energy (d) Consumption

Figure D.17: Least-Cost Path with $\psi = 3.64$

therefore, energy taxes must be higher in order to offset the large decline in extraction costs, relative to LGBGP levels (panel a). The path of innovation is essentially unaffected by the new calibration (panel b). Given the higher tax-inclusive energy price, this requires a slightly higher tax on energy efficiency R&D (panel a). The path of energy use that implements the least-cost path is also very similar to the baseline calibration (panel c). Since the externalities associated with energy use are increasing in ψ, the welfare effects of implementing the least-cost path are also larger. Panel (d) shows that consumption is higher than in the baseline calibration. The impact on the representative household from implementing the least-cost path is equivalent to a 1.5 percent increase in LFBGP consumption, which is smaller than the gain in the Cobb-Douglas model. Overall, the results demonstrate that the findings presented in the main text continue to hold with the significantly higher value of $\psi = 3.64$.



(a) Energy Use (b) R&D Allocations/Incentives

(c) Growth Determinants (d) Consumption

Figure D.18: Cost-less technology shock with exogenous energy prices

D.5 Rebound

D.5.1 CTS with Exogenous Energy Prices

Figure D.18 shows the results of a cost-less technology shock when energy prices are exogenous. The qualitative patterns are quite similar to those with endogenous energy prices, which are presented in Figure 9. The axes are the same in both figures for ease of comparison. The primary difference is that, after the initial shock, all variables in Figure D.18 converge monotonically to their new LFBGP levels. This demonstrates that the overshoot in Figure 9 is due to the endogenous energy price dynamics (see also the stability analysis in Appendix Section B.7.2). The model with exogenous energy prices still yields backfire in the long run.

D.5.2 Single-Period R&D Subsidy

In this section, I consider the case of unexpected, *single-period* 40 percent subsidy to energy efficiency R&D ($\eta^S_{2015} = 0.40$), which generates an immediate increase in $A_{E,t}$ of approximately 10 percent. Figure D.19 presents the results. The dynamics of energy use are similar to those following a CTS, but are more muted (panel a). In the long run, energy use returns to baseline levels and there is no backfire. Once again, the transition dynamics for energy use are slow. Flow energy use returns to baseline levels around 2100 and cumulative energy use returns to baseline levels in 2300.

Panel (b) presents the evolution of research incentives and allocations. In the period of the shock, $p_{E,t}/A_{E,t-1}$ remains at the baseline level, but the subsidy induces an increase in $R_{E,t}$ and $A_{E,t}$. This closely resembles the first period of the dynamics in the permanent subsidy case. In the next period, the subsidy disappears and $p_{E,t}/A_{E,t-1}$ is below the baseline level, causing $R_{E,t}$ to fall below the baseline level. Since subsidies have no futher impact on the incentives for R&D, the subsequent dynamics are similar to those in the cost-less technology shock case, and both variables converge back to their baseline levels.

Panel (c) shows the determinants of economic growth. As in the case of a permanent subsidy, the temporary subsidy pulls R&D inputs away from non-energy technology, causing $A_{N,t}$ to fall. There are now two offsetting forces on TFP, the increase in $A_{E,t}$ and the decrease in $A_{N,t}$. The net result is a slight decrease in TFP, which in turn causes a slight decrease in capital accumulation. Unlike the permanent subsidy case, TFP never rises above the LFBGP level. The small reduction in energy use following a temporary subsidy does not have a large enough impact on the price of energy to boost subsequent economic growth. As before, consumption is a constant fraction of final output (panel d).

D.5.3 Permanent Subsidy with Exogenous Energy Prices

Figure D.20 presents the results of a permanent subsidy when energy prices are exogenous. There are a couple of key differences with the endogenous price results presented in Figure 10. First, the trends in both flow and cumulative energy use are monotonic (panel a). The non-monotonicity in the baseline results was caused by the fall in extraction costs. Second, TFP and consumption decrease permanently. When energy prices are exogenous, there is no extraction cost externality, and the subsidy cannot boost economic growth and consumption via decreasing energy prices.

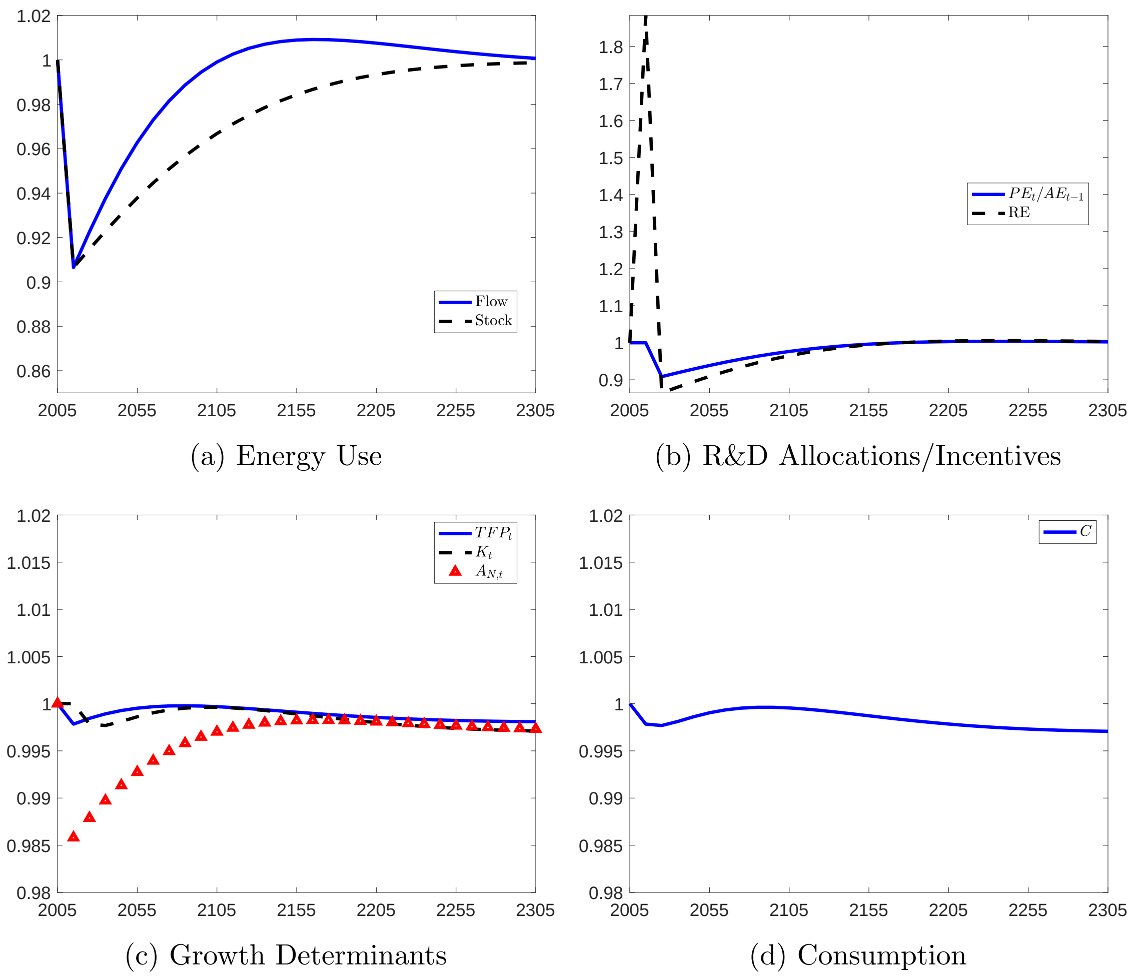


Figure D.19: Temporary Subsidy. Impact of a single period $\eta^S_{2015} = 40\%$ subsidy to energy efficiency R&D that causes $A_{E,t}$ to be 10 percent higher than the LFBGP level. All results are presented relative to the LFBGP.

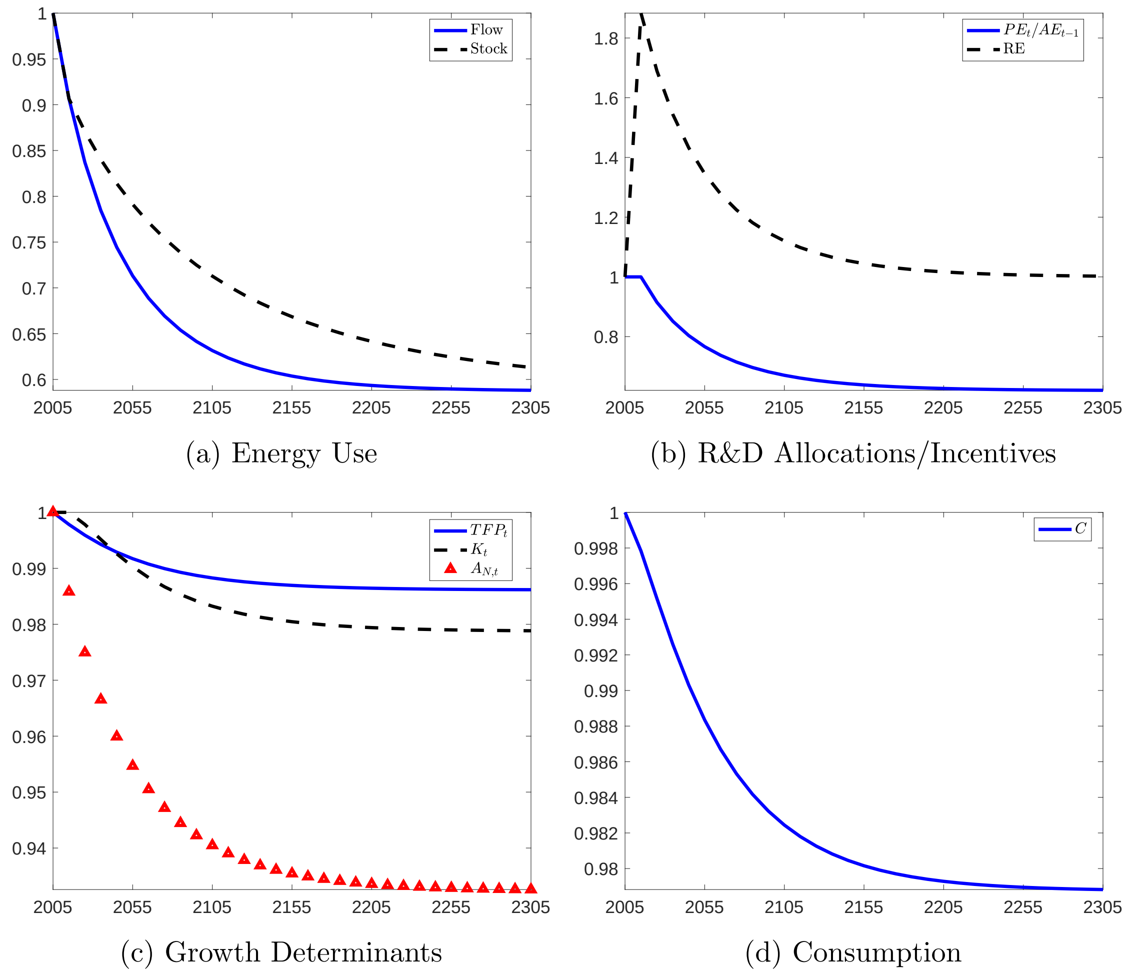


Figure D.20: Permanent Subsidy with exogenous energy prices.

E Additional Theoretical Results

E.1 Extension of Base Model

In this section, I consider an extension of the model that allows for labor reallocation between final good production and R&D, as well as the entry of new capital good producers. This extended model incorporates insights from 'second wave' endogenous growth theory (e.g., Peretto, 1998; Young, 1998; Howitt, 1999). As a result, it eliminates scale effects that are present in existing models of directed technical change (e.g., Acemoglu, 2002; Acemoglu et al., 2012; Hassler et al., 2021a). I show that the laissez-faire BGP of the extended model continues to explain the key patterns observed in U.S. data.

Consider the following extension of the aggregate production function:

$$Q_t = \int_0^{M_t} \min\left[(A_{N,t}(i)L_t/M_t)^{1-\alpha} X_t(i)^\alpha,\ A_{E,t}(i)E_t(i)\right] di, \tag{E.1}$$

where M_t gives the mass of capital good producers in operation at time t. This particular functional form is standard in the existing literature and eliminates the 'love of variety' in production. I will refer to L_t as production workers. To operate in period t, a capital good producer must hire φ^{-1} workers to cover fixed costs. This yields

$$M_t = \varphi F_t, \tag{E.2}$$

where F_t is the total number of workers hired to cover fixed costs. There is free entry into capital good production.

I assume

$$A_{J,t}(i) \;=\; \left[1+\eta_J R_{J,t}(i)^{1-\lambda}\right] A_{J,t-1}, \quad J = N, E, \tag{E.3}$$

where $A_{J,t} = \frac{1}{M_t}\int_0^{M_t} A_{J,t}(i)\, di$. As explained in footnote 15, this expression is equivalent to equation (8) in a decentralized equilibrium when $M_t = 1\ \forall t$. Here, $R_{J,t}(i)$ is the number of workers hired by firm i to improve technology J at time t. The new labor market clearing condition is given by

$$N_t = L_t + F_t + R_t\ \forall t, \tag{E.4}$$

where N_t is the size of the aggregate workforce. The key difference with the main text is that workers are now fully mobile across sectors. The size of the workforce grows at rate n. The utility function is now written in terms of N_t:

$$\{C_t\}_{t=0}^{\infty} \;=\; \operatorname{argmax} \sum_{t=0}^{\infty} \beta^t N_t \frac{\tilde{c}_t^{1-\sigma}}{1-\sigma}, \tag{E.5}$$

where $\tilde{c}_t = C_t/N_t$. The market clearing condition for capital is $K_t = \int_0^{M_t} X_t(i)di$. The remainder of the model is unchanged.

Using the same steps outlined in earlier Appendix Sections, it is straightforward to derive the following key expressions:

$$
\begin{aligned}
Y_t &= \left[1 - \frac{p_{E,t}}{A_{E,t}}\right] K_t^{\alpha} \left(A_{N,t} L_t\right)^{1-\alpha}, && \text{(E.6)}\\
w_t &= (1-\alpha) \left[1 - \frac{p_{E,t}}{A_{E,t}}\right] L^{-\alpha} A_{N,t}^{1-\alpha} K_t^{\alpha}, && \text{(E.7)}\\
\bar{\pi}_{X,t} &\propto r_t^{\frac{-\alpha}{1-\alpha}} A_{N,t} \left(L_t/M_t\right) \left[1 - \frac{p_{E,t}}{A_{E,t}}\right]^{\frac{1}{1-\alpha}}, && \text{(E.8)}\\
p_t^R &\propto r_t^{\frac{-\alpha}{1-\alpha}} \left(L_t/M_t\right) A_{N,t}(i) \left[1 - \frac{p_{E,t}}{A_{E,t}}\right]^{\frac{1}{1-\alpha}} \eta_N \bar{R}_{N,t}^{-\lambda} \frac{1}{1+g_{N,t}}, && \text{(E.9)}
\end{aligned}
$$

where $\bar{R}_{J,t} = M_t^{-1} \int_0^{M_t} R_{J,t}(i)di$ and $\bar{\pi}_{X,t}$ is the value of operating a capital good firm, conditional on the level of technology. Also, $\bar{R}_t = \bar{R}_{N,t} + \bar{R}_{E,t}$. I have applied the relevant market clearing conditions, and the fact that all capital good producers face an identical problem and make identical decisions. The latter implies that $\bar{R}_{J,t} = R_{J,t}(i)\ \forall i, J, t$. Compared to the equations for the baseline model, the only difference is the inclusion of M_t in the last two equations.

I will show that there exists a balanced growth path with constant expenditure shares, matching the data. On this BGP, a constant fraction of workers will be allocated to each occupation. I show this in two steps. First, I show that, conditional on constant labor allocations, the extended model reduces to the model presented in the main text. Thus, it has a BGP with constant expenditures shares for all factors including energy and is consistent with the balanced growth facts. Second, I show that this BGP is compatible with free mobility between occupations.

BGP assuming constant allocations. — Conditional on constant labor allocations (in shares), the BGP of the extended model is almost identical to the version presented in the main text. With constant allocations, L_t, M_t, and R_t grow at the same constant rate. Since R_t and M_t grow at the same rate, the average number of R&D workers per firm $\bar{R}_t$ is constant over time. This is also true in the base model, where both the number of firms and number of researchers are fixed. In terms of the return to R&D, the only difference with the main text is that L_t/M_t is constant, whereas the baseline model had only L_t, which was growing. The size of the labor force, however, has no impact on the relative return to improving the two types of technology. Taking $\bar{R}$ as given, therefore, the incentives for R&D

are essentially the same as in the main text,

$$\bar{R}_{E,t} = \frac{\sqrt{\frac{\tau_t p_{E,t}}{A_{E,t-1}}}\sqrt{\frac{1}{\alpha(1-\eta_t^S)}\left[\frac{\eta_E \bar{R}_{E,t}^{-\lambda}}{\eta_N(\bar{R}-\bar{R}_{E,t})^{-\lambda}} + \eta_E \bar{R}_{E,t}^{-\lambda} - \eta_E \bar{R}_{E,t}^{1-\lambda}\right] + (1+\eta_E \bar{R}_E^{1-\lambda})} - 1}{\eta_E \bar{R}_{E,t}^{-\lambda}}, \quad \text{(E.10)}$$

$$\bar{R}_{N,t} = \bar{R} - R_{E,t}, \quad \text{(E.11)}$$

with the only difference being that $\bar{R}$ replaces the normalized value of one. This immediately implies that $p_{E,t}/A_{E,t}$ is constant on the BGP, which gives a constant energy expenditure share.

With a constant energy expenditure share and growth rates of technology, the rest of the model is identical to the baseline case. Section 3.4 demonstrates that this model has a standard BGP that matches the usual balanced growth facts.

Wage growth on the BGP. — Now, I show that constant allocations are consistent with free mobility between sectors. In particular, I will show that wages in each sector grow at rate $g^*_{A_N}$, which is also the growth rate of output per capita. Equations (E.6) and (E.7) are identical to the baseline model. They imply that the wages of production workers grow at the rate of output per capita. The intuition is straightforward as these are standard equations from the neoclassical growth model once $p_{E,t}/A_{E,t}$ is constant.

Now, I turn to the wages for researchers. On a BGP, R&D allocations, technology growth rates, and the return to capital will all be constant. With constant allocations, L_t/M_t is constant. Putting these together, it is immediate from (E.9) that the wage paid to scientists, p^R_t also grows at $g^*_{A_N}$ on the BGP.

Finally, payments to fixed cost workers are determined by the free entry condition. In particular, wages paid to fixed cost workers are proportional to the value of operating a capital good firm conditional on technology ($\bar{\pi}_{X,t}$) minus R&D costs,

$$\begin{aligned} w^F_t &= \frac{1}{\varphi}\left(\bar{\pi} - p^R_t\right) && \text{(E.12)} \\ &\propto A_{N,t}(i), && \text{(E.13)} \end{aligned}$$

which also grows at $g^*_{A_N}$ on the BGP. So, the growth rate of wages is the same for all occupations. The labor allocation is then determined by the market clearing condition (E.4) and the free mobility condition, $w_t = w^F_t = p^R_t$.

So, the extended model explains the same BGP facts while allowing for labor reallocation between final good production and R&D. Following the existing quantitative literature on the macroeconomics of climate change – e.g., Acemoglu et al. (2016) and Fried (2018) – the analyses conducted in this paper use the baseline model, which does not allow for this reallocation. The extended model demonstrates that the core intuition of the baseline model holds in this richer setting.

E.2 R&D Spillovers and Differential Productivity Growth

Data on energy use and productivity growth indicate that growth in energy efficiency is faster than overall technology growth ($g^*_{A_E} > g^*_{A_N}$). Matching this fact places restrictions on the functional form for (8), the law of motion for technology. In particular, it rules out R&D spillovers between technologies and semi-endogenous growth specifications.

To see this, consider the following alternate R&D specification

$$A_{E,t}(i) = \left[1 + \eta_E R_{E,t}(i)^{1-\lambda}\right] A_{E,t-1}^{1-\phi} A_{N,t-1}^{\phi}, \tag{9'}$$

where $\phi \in (0,1)$ and the law of motion for $A_{N,t}(i)$ has a symmetric form. As explained in footnote 15, this expression is equivalent to equation (8) in a decentralized equilibrium when $\phi = 0$. The degree of spillovers is decreasing in ϕ, and $\phi = 0$ corresponds to the case of no spillovers.

Following the same steps as in Appendix Section B.2, the research arbitrage condition is

$$1 = \frac{\alpha\left[1 - \frac{p_{E,t}}{A_{E,t}}\right]}{\frac{p_{E,t}}{A_{E,t}}} \cdot \frac{\eta_N R_N^{-\lambda}}{\eta_E R_E^{-\lambda}} \cdot \frac{A_{E,t} A_{N,t-1}^{1-\phi} A_{E,t-1}^{\phi}}{A_{N,t} A_{E,t-1}^{1-\phi} A_{N,t-1}^{\phi}}. \tag{E.14}$$

Clearly, the LHS of this equation is constant. The first two terms on the RHS are the same as in the baseline model. To match the trendless energy expenditure share, the first term on the RHS must be constant on the BGP. Similarly, for productivities to grow at a constant rate, R&D allocations must be constant on the BGP, which implies that the second term is constant.

It is straightforward to rewrite the last term as $\frac{1+g_{E,t}}{1+g_{N,t}} \cdot \frac{A_{E,t-1}^{2\phi}}{A_{N,t-1}^{2\phi}}$. Since technological progress is constant on the BGP, this term could only be constant if $\frac{A_{E,t-1}}{A_{N,t-1}}$ is constant or $\phi = 0$. As explained above, the former option is inconsistent with the data. So, the data are inconsistent with a model that includes research spillovers.

It is also straightforward to show that the same limitation holds for a semi-endogenous specification. In this case,

$$A_{E,t}(i) = \left[1 + \eta_E R_{E,t}(i)^{1-\lambda}\right] A_{E,t-1}^{\phi}, \quad \phi \leq 1 \tag{9''}$$

and

$$1 = \frac{\alpha\left[1 - \frac{p_{E,t}}{A_{E,t}}\right]}{\frac{p_{E,t}}{A_{E,t}}} \cdot \frac{\eta_N R_N^{-\lambda}}{\eta_E R_E^{-\lambda}} \cdot \frac{A_{E,t} A_{N,t-1}^{\phi}}{A_{N,t} A_{E,t-1}^{\phi}}. \tag{E.15}$$

Following the same steps as above rules out the case where $\phi < 1$.

Appendix References

Acemoglu, D. (2002): "Directed technical change," *Review of Economic Studies*, 69, 781–809.

Acemoglu, D., P. Aghion, L. Bursztyn, and D. Hemous (2012): "The environment and directed technical change," *American Economic Review*, 102, 131–66.

Acemoglu, D., U. Akcigit, D. Hanley, and W. Kerr (2016): "Transition to clean technology," *Journal of Political Economy*, 124, 52–104.

Adjemian, S., H. Bastani, M. Juillard et al. (2011): "Dynare: Reference Manual Version 4," Dynare Working Papers 1, CEPREMAP.

Barrage, L. (2013): "Sensitivity Analysis for Golosov, Hassler, Krusell, and Tsyvinski (2013): Optimal Taxes on Fossil Fuel in General Equilibrium," *Supplementary Material, Econometrica*, to: Golosov, M., J. Hassler, P. Krusell, and A. Tsyvinski (2014)"Optimal Taxes on Fossil Fuel in General Equilibrium," *Econometrica*, 82(1): 41-88.

——— (2020): "Optimal dynamic carbon taxes in a climate-economy model with distortionary fiscal policy," *Review of Economic Studies*, 87, 1–39.

Dietz, S., A. Bowen, B. Doda, A. Gambhir, and R. Warren (2018): "The economics of 1.5°C climate change," *Annual Review of Environment and Resources*, 43, 455–480.

Fried, S. (2018): "Climate policy and innovation: A quantitative macroeconomic analysis," *American Economic Journal: Macroeconomics*, 10, 90–118.

Fuss, S., J. G. Canadell, G. P. Peters et al. (2014): "Betting on negative emissions," *Nature Climate Change*, 4, 850–853.

Gasser, T., C. Guivarch, K. Tachiiri, C. Jones, and P. Ciais (2015): "Negative emissions physically needed to keep global warming below 2 C," *Nature Communications*, 6, 1–7.

Golosov, M., J. Hassler, P. Krusell, and A. Tsyvinski (2014): "Optimal taxes on fossil fuel in general equilibrium," *Econometrica*, 82, 41–88.

Hart, R. (2019): "To everything there is a season: Carbon pricing, research subsidies, and the transition to fossil-free energy," *Journal of the Association of Environmental and Resource Economists*, 6, 349–389.

Hassler, J., P. Krusell, and C. Olovsson (2021): "Directed technical change as a response to natural-resource scarcity," *Journal of Political Economy*, 129, 3039–3066.

Hassler, J., P. Krusell, and A. A. Smith Jr (2016): "Environmental macroeconomics," in *Handbook of Macroeconomics* Volume 2: Elsevier, 1893–2008.

Heal, G. (2017): "Reflections: What would it take to reduce US greenhouse gas emissions 80 percent by 2050?" *Review of Environmental Economics and Policy*, 11, 319–335.

Howitt, P. (1999): "Steady endogenous growth with population and R&D inputs growing," *Journal of Political Economy*, 107, 715–730.

McGlade, C. E. (2014): *Uncertainties in the outlook for oil and gas*, Ph.D. dissertation, University College London.

McGlade, C., and P. Ekins (2015): "The geographical distribution of fossil fuels unused when limiting global warming to 2°C," *Nature*, 517, 187–190.

Nordhaus, W. D., and J. Boyer (2000): *Warming the world: Economic models of global warming*: MIT press.

Peretto, P. F. (1998): "Technological change and population growth," *Journal of Economic Growth*, 3, 283–311.

Rogner, H.-H., L. A. LA, R. F. Aguilera et al. (2012): "Energy Resources and Potentials," in *Global energy assessment: Toward a sustainable future* ed. by Johansson,

T. B., Patwardhan, A. P., Nakićenović, N., and Gomez-Echeverri, L.: Cambridge University Press.

Williams, J. H., B. Haley, F. Kahrl, J. Moore, A. D. Jones, M. S. Torn, H. McJeon et al. (2014): "Pathways to deep decarbonization in the United States," *The US Report of the Deep Decarbonization Pathways Project of the Sustainable Development Solutions Network and the Institute for Sustainable Development and International Relations, Energy and Environmental Economics, San Francisco, CA, accessed Apr*, 23, 2016.

Young, A. (1998): "Growth without scale effects," *Journal of Political Economy*, 106, 41–63.