Digital Currency and Banking-Sector Stability

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Abstract

Digital currencies provide a potential form of liquidity competing with bank deposits. We introduce stable digital currency into a macro model with a financial sector in which financial frictions generate endogenous systemic risk and instability. In the model, digital currency is fully integrated into the financial system and depresses bank deposit spreads, particularly during crises, which limits banks' ability to recapitalize following losses. As a result, the probability of the banking sector being in crisis or distressed states can grow significantly with the introduction of digital currency. While banking-sector stability suffers, asset price volatility decreases, and household welfare can improve significantly. Despite the potential welfare gains, our theoretical results suggest that financial frictions may limit the potential benefits of digital currencies. The optimal level of digital currency may be below what would be issued in a competitive environment.

Keywords: Financial stability, Macroeconomic instability, Financial frictions, CBDC, Stablecoins.

JEL classification: E44, E52, E58, G01, G12, G20, G21.

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1 Introduction

Digital currencies could become an alternative to traditional money. Private suppliers are trying to design digital currencies with stable values that can function as reliable means of payment. Central banks are also considering whether to issue cryptocurrency. On the one hand, many privately issued stablecoins are far from stable in their current implementations,[1] and a central bank digital currency (CBDC) has the potential to generate fragility and runs (European Central Bank, 2020; Federal Reserve, 2022) or widespread disintermediation as depositors move funds out of bank deposits and into CBDCs (Fernández-Villaverde, Sanches, Schilling, and Uhlig, 2021). On the other hand, there is the potential for a new "steady state" in which digital currencies are fully integrated into the financial sector, competing with bank deposits as an alternative form of liquidity and providing stable means of payment. In this case, digital currencies might affect banks' investment activities without leading to complete disintermediation. More broadly, there remains uncertainty about how digital currencies, whether issued publicly or privately, would affect the stability of the traditional financial sector.

In this paper, we consider the financial stability consequences of digital currency should it become fully integrated into the financial sector, whether in the form of CBDC or truly stable stablecoins. We suppose that digital currencies and bank deposits coexist, and households hold both digital currencies and bank deposits within their portfolio of liquid claims. Accordingly, we take as given a degree of "disintermediation" caused by digital currencies. Instead of focusing our analysis on distintermediation or the potential for runs, we focus on financial instability caused by systemic *deleveraging*, leading to fire sales, and *undercapitalization* of the banking sector.

Our primary contribution to this debate is a quantitative model in which banks are subject to financial frictions regarding equity issuance, and cheap funding via deposit spreads help stabilize the system. We consider a continuous-time stochastic general equilibrium model in which financial frictions endogenously create inefficient instability and systemic risk, building on Brunnermeier

[1] Gorton et al. (2022) find empirically that stablecoins currently carry an *inconvenience yield*: investors require additional returns in exchange for holding stablecoins compared to traditional risk-free assets, whereas investors are willing to forego returns in order to hold fiat currency (i.e., fiat currency carries a *convenience* yield). d'Avernas et al. (2022) show theoretically that only fiat-backed coins are truly stable.

and Sannikov (2014). We assess the consequences of digital currency for financial stability and welfare by investigating the impact of digital currency on the amplification of shocks, nonlinear dynamics, and systemic risk, which are central features of the framework in Brunnermeier and Sannikov (2014). This approach contrasts with the previous literature studying CBDCs because we focus on financial fragility caused by low levels of bank equity and systemic deleveraging rather than bank runs or disintermediation.

In the model, banks invest in productive capital, but due to frictions, banks' marginal source of funding is typically risk-free deposits because issuing equity is costly. As a result, banks invest more when they have more equity, and capital is allocated more efficiently when banks are well-capitalized. To this setting we add digital currencies as a substitute form of liquidity that competes with bank deposits. We solve for the global dynamics of the economy to assess the impact of digital currency on bank profitability and thus on bank equity growth.

The main mechanisms of the model are as follows. Digital currencies decrease the spreads on bank deposits, thus affecting the ability of banks to rebuild equity following losses and the ex-ante investment decisions (e.g., leverage) of banks in good times. The effect on spreads is greatest in crises, precisely when banks are in most need of being able to recapitalize quickly. As a result, digital currencies can have significant adverse consequences for the frequency and duration of good and bad outcomes.

Our main positive result is that digital currency can significantly harm the stability of the banking sector: times of distress and crisis increase monotonically with the supply of digital currency, and bank valuation decrease substantially. At the welfare-maximizing level of digital currency, the probability of crises in our benchmark calibration doubles from 3% to 5.97%, output losses in crises increase from 8% to 8.78%, and bank valuations decrease by 6.8%. Nonetheless, there are some silver linings for financial *markets*: asset price volatility generally decreases and asset prices increase. In terms of welfare, household welfare can increase significantly despite the decrease in financial stability. These welfare consequences are large. In our benchmark calibration, we find plausible welfare gains on the order of 2% in terms of consumption-equivalent. Thus, our model suggests that there is a robust range of digital currency issuance in which the welfare-stability trade-off is in favor of digital currency, even when digital currency is costly to issue.

While financial frictions do not completely overturn the potential welfare gains from digital currency, they do affect the social optimum. We find that the optimal level of digital currency is lower with financial frictions than without. Not only that, the welfare-maximizing level of digital currency may be less than what would be provided by profit-maximizing issuers in a competitive market. Thus, financial frictions may provide an additional rationale for regulating stablecoins issuance even in the best-case scenario when they are truly stable and run-proof.

Related Literature. There is a growing literature studying the impact of digital currencies on the financial sector, especially focusing on CBDCs. One strand of the literature focuses on the impact of CBDC on normal (non-crisis) periods (e.g., Keister and Sanches, 2021; Williamson, 2022). Another strand considers how CBDC affects fragility in the sense of affecting runs within a Diamond and Dybvig (1983) setting, i.e., providing new incentives for depositors to withdraw or run on banks. Notable contributions include Skeie (2021); Williamson (2021); Keister and Monnet (2022); Ahnert et al. (2020). There is also an important emphasis on the potential for disintermediation within the traditional banking sector (Fernández-Villaverde, Sanches, Schilling, and Uhlig, 2021). These papers echo the concerns of policymakers: a recent report by the European Central Bank (2020) states "in crisis situations, when savers have less confidence in the whole banking sector, liquid assets might be shifted very rapidly from commercial bank deposits to the digital euro," and a report from the Federal Reserve (2022) worries that "CBDC could make runs on financial firms more likely or more severe."

The effect of digital currencies on fragility in the sense of *runs* is clearly important to understand. Nonetheless, our paper takes a different approach, considering an environment without runs in this sense. In contrast to the existing literature, our emphasis is on fragility caused by systemic *deleveraging*, leading to fire sales and an undercapitalized banking sector. We consider a scenario in which financial stability relates to the level of equity in the banking sector, which determines the degree of intermediation. Digital currencies and bank deposits coexist, and we take as given a degree of disintermediation caused by digital currencies in the sense that households hold both digital currencies and bank deposits within their portfolio of liquid claims. Our paper broadly provides insight about how any substantial competition with deposits has the potential to create instability in the banking sector, with a primary application to digital currency, which has the potential to be one such form of competition.

Methodologically, our paper follows the stochastic continuous-time macro literature pioneered by Brunnermeier and Sannikov (2014, 2015, 2016a) and He and Krishnamurthy (2012, 2013, 2019), who analyze the nonlinear global dynamics of economies with financial frictions, building on seminal results from Kiyotaki and Moore (1997) and Bernanke, Gertler, and Gilchrist (1999). Several papers have introduced money and liquidity provision within this literature. Moreira and Savov (2017) focus on the relationship between macroeconomic instability and the transformation of risky assets into liquid securities by shadow banks. Phelan (2016) considers the role of banks as providers of liquidity via deposits and notes a trade off between financial stability and economic growth because financial intermediation leads to more efficient investment. Drechsler et al. (2018) develop an asset-pricing model in which banks hold liquidity to insure deposits against funding risks. Di Tella (2020) studies the real effects of money through effects on risk premia and idiosyncratic risk sharing. Chen and Phelan (2022a) study the consequences of financial frictions for aggregate bank equity and welfare when banks primarily provide liquidity services, and Chen and Phelan (2022b) study the role of monetary policy to address stability concerns.[2]

A recent closely related paper is Burlon et al. (2022), who consider disintermediation through deposit substitution caused by CBDC in a quantitative DSGE model calibrated to the euro area. In their model, the financial frictions are collateral constraints facing household and firm borrowers, who each borrow from banks that are subject to capital adequacy constraints. Similar to our paper, they find that there are welfare gains despite bank disintermediation. In contrast to our paper, they use a linearized model with second-order approximations for welfare from CBDC, whereas we consider financial stability and crises in a nonlinear model and consider both publicly and privately produced digital currency.[3] Whited et al. (2022) show that CBDC need not reduce bank lending unless frictions and synergies bind deposits and lending together. In their model, CBDC

[2]Additional work in the continuous-time macro-finance literature include Adrian and Boyarchenko (2012), Maggiori (2017), Di Tella (2017), and Van der Ghote (2019, 2021).

[3]For the broader issues relating to CBDC design and how CBDC might relate to deposits or other digital currencies, see Ahnert et al. (2022a,b) who specifically highlight privacy concerns.

has a much smaller impact on bank lending because banks can replace a large fraction of any lost deposits with wholesale funding.

2 Model

The economy is populated by households and banks, which are owned by households. Output is produced using a single factor of production, a Lucas tree. Bank deposits and digital currencies earn a liquidity yield, and thus banks have an advantage for financing the tree. The financial friction is costly equity issuance. Our model borrows from Chen and Phelan (2022a,b), which build on Brunnermeier and Sannikov (2014) and Phelan (2016).

2.1 Technology, Environment, and Markets

Time is continuous and infinite, and aggregate productivity shocks follow a Brownian motion.

Output Output in the economy is produced by a tree with a stochastic dividend Y_t. Because the tree is the only source of output in the economy, we refer to it interchangeably as "the risky asset" or as "capital." The tree's dividend Y_t evolves according to equation (1),

$$\frac{dY_t}{Y_t} = g_{y,t}\,dt + \sigma\,dW_t, \tag{1}$$

where W_t is an exogenous Brownian aggregate shock, σ is the exogenous fundamental volatility, and $g_{y,t}$ is the growth rate. We suppose that banks induce a higher exogenous growth rate when holding the tree, which captures gains from intermediation, i.e., $g_b \geq g_h$ with banks and households denoted by b and h. Let ψ_t denote the fraction of capital held by banks in equilibrium. Then the aggregate growth rate is

$$g_{y,t} = \psi_t g_b + (1-\psi_t) g_h$$

and is therefore an endogenous combination of the exogenous growth rates.

Asset Price and Investment Returns The risky asset trades in a perfectly competitive market with real asset price Q_t. We postulate that its law of motion takes the form

$$\frac{dQ_t}{Q_t} = \mu_{Q,t}\,dt + \sigma_{Q,t}\,dW_t, \tag{2}$$

which will be determined endogenously in equilibrium. The return to owning the risky asset includes the dividend yield of the output produced $(1/Q_t)$ and the capital gains on the value of the tree holdings. By Ito's Lemma, the rate of return for agent i is given by

$$dr_{i,t} = \left(\frac{1}{Q_t} + g_i + \mu_{Q,t} + \sigma\sigma_{Q,t}\right)dt + (\sigma + \sigma_{Q,t})dW_t. \tag{3}$$

The volatility of returns on investments is $\sigma + \sigma_{Q,t}$, which includes fundamental risk σ and endogenous price risk $\sigma_{Q,t}$.

Bonds, Deposits, and Digital Currency Bank equity shares trade in a perfectly competitive market, where $dr_{e,t}$ denotes the return on bank equity.

There are several markets for risk-free assets in this economy, all of which are in zero net supply: government bonds, bank deposits, and digital currency (DC). Government bonds pay the real risk-free rate $dr_{f,t} = r_{f,t}dt$, determined endogenously by households' stochastic discount factor. Deposits and digital currency provide liquidity services in addition to safe storage and therefore pay returns below the risk-free rate. Bank deposits pay $dr_{d,t} = r_{d,t}dt$, and digital currency pays $dr_{dc,t} = r_{dc,t}dt$, both determined in equilibrium.[4] Let

$$DS_t \equiv r_{f,t} - r_{d,t}, \qquad DCS_t \equiv r_{f,t} - r_{dc,t}$$

denote the spreads on bank deposits and digital currency.

We consider public and private provision of digital currency. For the purposes of our paper, the two ways of issuing digital currency are generally equivalent. The government issues two forms of debt, bonds and digital currency (i.e., CBDC). Let $\mathbb{B}_t$ and $\mathbb{CBDC}_t$ denote the real supply of

[4] In equilibrium all returns will be continuous with finite paths of variation, enabling this form.

government bonds and CBDC. For tractability, we suppose that both bonds and CBDC grow at the rate of capital growth $g_{y,t}$ so that total government debt is a constant fraction of output Y_t. Thus, we can write total debt $\mathbb{D}_t = \mathbb{B}_t + \mathbb{CBDC}_t$ as

$$\mathbb{D}_t = q^D Y_t.$$

We suppose that issuing a digital currency entails a flow intermediation cost of κ goods per unit. The government's budget constraint is therefore given by

$$\mathbb{B}_t(r_{f,t} - g_{y,t}) + \mathbb{CBDC}_t(r_{dc,t} - g_{y,t} + \kappa) = T_t, \tag{4}$$

where T_t is total taxes. Because we are not interested in the fiscal consequences of a CBDC, we suppose that taxes are collected lump-sum from households (there are no government expenditures apart from servicing the debt). Note, however, that, due to liquidity services, CBDC will pay an interest rate below the risk-free rate thus *improving* the government's budget deficit.

We also consider private provision of digital currency. Given the scope of this paper, we only consider truly stable digital currencies: all digital currency must be completely backed by government assets (d'Avernas et al., 2022). We therefore assume a firm (technology) that holds government bonds, earning $dr_{f,t}$, to issue digital currency paying $dr_{dc,t}$, and pays the cost κ per unit. Thus, the flow profits $T_{p,t}$ from issuing $\mathbb{DC}_t$ units of digital currency are

$$\mathbb{DC}_t(DCS_t - \kappa) = T_{p,t},$$

It is instructive to rewrite the government budget constraint as follows:

$$\begin{aligned}\mathbb{B}_t(r_{f,t} - g_{y,t}) + \mathbb{CBDC}_t(r_{f,t} - g_{y,t}) - \mathbb{CBDC}_t(r_{f,t} - r_{dc,t} - \kappa) &= T_t,\\ \mathbb{D}_t(r_{f,t} - g_{y,t}) - \mathbb{CBDC}_t(DCS_t - \kappa) &= T_t,\end{aligned}$$

Note that the term $\mathbb{CBDC}_t(DCS_t - \kappa)$ is precisely the flow profits of privately issuing $\mathbb{CBDC}_t$ units of digital currency backed by government bonds. Publicly issued digital currency is implicitly backed by government bonds. For a given level of total debt $\mathbb{D}_t$, issuing CBDC instead of bonds simply changes the composition of the government balance sheet. In reality, a CBDC would likely be issued by a central bank that backs the digital currency with holdings of government debt. Hence, equation (4) is the consolidated budget constraint for the government.

Going forward we do not specify whether digital currency is publicly or privately issued; we simply let $\mathbb{DC}_t$ denote the total supply of digital currency. Later we consider the private incentives to issue digital currency. Let m denote the fraction of government liabilities used to back digital currency (publicly or privately). Then we have $\mathbb{DC}_t = mq^D Y_t$ and the total intermediation cost per unit of Y_t is $\iota \equiv \kappa m q^D$. Our main question of interest is how m affects equilibrium.

In sum, bank deposits are issued by banks, which invest in the tree, and thus deposits are backed by real (risky) investments. Digital currency, in contrast, is backed by risk-free government liabilities, which are ultimately backed by taxes, but issuing digital currency entails a cost κ.

2.2 Households, Banks, and Portfolio Choice

There is a continuum of risk-averse households denoted by $h \in [0,1]$ with initial wealth $w_{h,0}$ and a continuum of banks, denoted by $b \in [0,1]$, with initial book value ("equity") $n_{b,0}$.

Households and Their Portfolio Choice Households have log utility over consumption and "liquidity-in-the-utility-function" over liquidity services ℓ. Lifetime utility V_t is given by

$$V_t = \mathbb{E}_t \left[\int_t^\infty e^{-r(\tau - t)} \left(\log(c_{h,\tau}) + \beta \log\left(\ell_{h,\tau}\right) \right) d\tau \right], \tag{5}$$

where r is the discount rate, $c_{h,\tau}$ is flow consumption, $\ell_{h,\tau}$ is liquidity services, and β is a preference parameter. Let $v(\ell) \equiv \beta \log(\ell)$ denote liquidity utility for later use. Liquidity services are derived from holding bank deposits $depo_{h,t}$ and digital currency $dccy_{h,t}$ according to

$$\ell_{h,t} = \left(\left(\frac{depo_{h,t}}{w_{h,t}} \right)^{\frac{\varepsilon-1}{\varepsilon}} + \delta \left(\frac{dccy_{h,t}}{w_{h,t}} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

The parameter ε is the elasticity of substitution between bank deposits and digital currency, and δ is a preference parameter determining the relative attractiveness of the two types of liquidity. Liquidity services are scaled by wealth to preserve homogeneity. A preference for liquidity arises in many models. Because the focus of our paper is how digital currencies affect financial stability and not on the microfoundations of the value of money, we model the value of bank deposits directly in the utility function for convenience, in essence taking as given the microfoundations for the preference for liquidity (see Diamond and Dybvig (1983), Gorton and Pennacchi (1990), Lagos and Wright (2005), and Gali (2008)).

Let $x_t = (x_{y,t}, x_{f,t}, x_{d,t}, x_{dc,t}, x_{e,t})$ be household's portfolio weight on capital, risk-free bonds, deposits, digital currency, and bank equity respectively. Let $dr_t = (dr_{y,t}, dr_{f,t}, dr_{d,t}, dr_{dc,t}, dr_{e,t})$ be the vector of returns. Formally, households solve the problem

$$\max_{x_t \geq 0, c_{h,t}} \mathbb{E}_t \left[\int_t^{\infty} e^{-r(\tau - t)} \left(\log(c_{h,\tau}) + \beta \log\left(\ell_{h,\tau}\right) \right) d\tau \right],$$

subject to

$$\frac{dw_{h,t}}{w_{h,t}} = x_t \cdot dr_t + \left(\mathscr{T}_{h,t} - \frac{c_{h,t}}{w_{h,t}} \right) dt, \tag{6}$$

$$\ell_{h,t} = \left((x_{d,t})^{\frac{\varepsilon-1}{\varepsilon}} + \delta (x_{dc,t})^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{7}$$

$$w_{h,t}, x_{y,t} \geq 0, \tag{8}$$

$$x_{y,t} + x_{f,t} + x_{d,t} + x_{dc,t} + x_{e,t} = 1, \tag{9}$$

where $\mathscr{T}_{h,t}$ denotes all transfers received by households normalized by their net worth. Agents cannot short capital $x_{y,t} \geq 0$. Equation (6) characterizes the evolution of households' total wealth. Transfers include dividends from banks and any payouts (taxes) from the government. Let the evolution of $c_{h,t}$ follow the finite diffusion

$$\frac{dc_{h,t}}{c_{h,t}} = \mu_{ch,t}\, dt + \sigma_{ch,t}\, dW_t.$$

Let $\sigma_{e,t}$ denote the volatility of bank equity returns. Proposition 1 characterizes households' optimal decisions.

Proposition 1. *Households choose x_t and $c_{h,t}$ such that*

(i) $c_{h,t} = rw_{h,t}$,

(ii) $DS_t = rv'(\ell_{h,t})\frac{\partial \ell}{\partial x_{d,t}}$,

(iii) $DCS_t = rv'(\ell_{h,t})\frac{\partial \ell}{\partial x_{dc,t}}$,

(iv) $\mathbb{E}[dr_{e,t}] - r_{f,t} = \sigma_{ch,t}\sigma_{e,t}$,

(v) $\mathbb{E}[dr_{h,t}] - r_{f,t} \leq \sigma_{ch,t}(\sigma + \sigma_{Q,t})$, *with equality if and only if* $x_{y,t} > 0$,

where DS_t and DCS_t are the deposit spread and the digital currency spread, respectively.

Since banks are shareholder-maximizing firms rather than separate agents, households internalize the value of future bank dividends in their consumption policy. Therefore, households consume a fraction r of their total wealth and hold positive quantities of capital if the expected excess returns equal the covariance of their consumption with returns. The spreads on deposits and digital currencies relative to the risk-free rate are determined by the marginal utility of liquidity $v'(\ell)$.

Banks and Their Portfolio Choice Banks invest in capital and issue deposits. Banks are owned by households, who choose dividend payouts, the level of deposits, the level of liquid reserves, and the portfolio weight on capital used by banks. Because of un-modeled financial frictions, banks are subject to two constraints. First, it is costly for banks to issue equity; raising one unit of equity costs $1+\gamma$ units (i.e., γ is the marginal issuance cost). Second, the value of banks' assets minus liabilities $n_{b,t}$ cannot become negative (bankruptcy).[5]

Banks' maximize the present value of dividends discounted according to the households' discount factor, subject to their constraints. Dividends are discounted by households' stochastic discount factor ξ_t (SDF). Since households have log utility, the stochastic discount factor is $\xi_t = e^{-rt}c_{h,t}^{-1}$. Due to banks' productivity advantage and ability to issue deposits, banks will issue deposits to hold capital, but to avoid bankruptcy banks will never finance their portfolio with only deposits.

[5] Acharya et al. (2011) provide empirical support that bank equity is "sticky": during the financial crisis, banks raised capital in the form of debt, not common equity. Adrian and Shin (2010, 2014) provide evidence that the predetermined balance sheet variable for banks is equity, not assets.

We interpret banks in our model as representing financial intermediaries beyond simply deposit-taking commercial banks but also capturing other bank-like financial intermediaries whose liabilities provide money-like services. Hence, the "banking sector" in our model is meant to capture this portion of the financial sector as a whole. Empirically, He, Kelly, and Manela (2017) find that the main buyers of asset sales from investment banks and hedge funds are commercial banks. In our model, we would interpret such trades as trades between banks, not between banks and households. Importantly, because we are interested in how the provision of digital currency would affect financial stability, we suppose that banks cannot issue digital currency backed by government bonds; the bank technology allows them to use real assets to issue deposits. We will return to this issue later.

Let $x_{b,t}$ denote the fraction of equity $n_{b,t}$ held in capital. Formally, banks solve the problem

$$\max_{x_{b,\tau}\geq 0, d\zeta_{b,\tau}} U_t = \mathbb{E}_t\left[\int_t^\infty \frac{\xi_\tau}{\xi_t} d\zeta_{b,\tau}\right],$$

subject to the equity issuance friction and two constraints on the evolution of their (book) equity

$$\frac{dn_{b,t}}{n_{b,t}} = dr_{d,t} + x_{b,t}\left(dr_{b,t} - dr_{d,t}\right) - \frac{d\zeta_t}{n_{b,t}} - \mathscr{T}_{b,t}\,dt, \tag{10}$$

$$n_{b,t}, x_{b,t} \geq 0. \tag{11}$$

Banks pay interest on their deposits, earn a return from their capital holdings, and pay dividends at rate $d\zeta_{b,t}$. They also pay transfers $\mathscr{T}_{b,t}$ per unit equity, which we use primarily as a modeling device to calibrate the model.

By homogeneity and price-taking, the maximized value of a bank with equity $n_{b,t}$ can be written

$$\theta_t n_{b,t} \equiv \max_{\{x_{b,\tau}\geq 0, d\zeta_{b,\tau}\}} \mathbb{E}_t\left[\int_t^\infty \frac{\xi_\tau}{\xi_t} d\zeta_{b,\tau}\right]. \tag{12}$$

The marginal value of equity θ_t equals 1 plus the multiplier on the equity-issuance constraint and reflects the aggregate condition of the financial sector. Proposition 2 characterizes banks' optimality conditions, and in particular describes the equilibrium diffusion for θ_t.

Proposition 2. *Assume θ_t follows the finite diffusion*

$$\frac{d\theta_t}{\theta_t} = \mu_{\theta,t}\,dt + \sigma_{\theta,t}\,dW_t, \tag{13}$$

with $\sigma_{\theta,t} \leq 0$. Then $\theta_t n_t$ represents the maximal future expected payoff that a bank with book value n_t can attain, and $\{x_{b,t}, d\zeta_t\}$ is optimal if and only if

(i) $\theta_t \in [1, 1+\gamma]\ \forall t$,

(ii) $d\zeta_t > 0$ *only when* $\theta_t = 1$, $d\zeta_t < 0$ *only when* $\theta_t = 1+\gamma$, *and* $d\zeta_t = 0$ *otherwise,*

(iii) $\mu_{\theta,t} = DS_t + \sigma_{ch,t}\sigma_{\theta,t} + \mathscr{T}_{b,t}$,

(iv) $\mathbb{E}[dr_{b,t}] - r_{d,t} \leq (\sigma_{ch,t} - \sigma_{\theta,t})(\sigma + \sigma_{Q,t})$, *with strict equality when* $x_{b,t} > 0$,

(v) The transversality condition $\mathbb{E}[\xi_t \theta_t n_t] \to 0$ *holds under* $\{x_{b,t}, d\zeta_t\}$.

Moreover, bank shares are priced such that $\mathbb{E}[dr_{e,t}] = \mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t}\sigma_{nb,t}$ *and* $\sigma_{e,t} = \sigma_{\theta,t} + \sigma_{nb,t}$.

Banks will not pay dividends when $\theta_t \geq 1$; θ_t can never be less than one because banks can always pay out the full value of equity, guaranteeing a value of at least $n_{b,t}$. Banks will raise equity when $\theta_t = 1+\gamma$; θ_t can never be great than $1+\gamma$ because banks can always immediately raise equity and pay a dividend. Banks demand a risk premium that is higher than households' because although banks discount dividends using the household SDF, banks are also concerned about their solvency due to costly equity issuance.

2.3 Equilibrium

A competitive equilibrium is characterized by the market price for the risky asset, together with portfolio allocations and consumption decisions such that given prices, agents optimize and markets clear. Due to equity issuance frictions, banks' decisions depend on their level of equity, and so equilibrium depends on banks' equity levels and monetary policy has scope to affect equilibrium.

Equilibrium Dynamics We solve for the global equilibrium dynamics using the methods in Brunnermeier and Sannikov (2014). Define $N_{b,t} = \int n_{b,t}\,db$ as aggregate bank equity. Since the tree dividend grows geometrically and the bank problem is homogeneous, the equilibrium state variable of interest is aggregate bank equity as a fraction of the total value of the tree:

$$\eta_t \equiv \frac{N_{b,t}}{Q_t Y_t}.$$

Equilibrium consists of a law of motion for η_t and asset allocations and prices as functions of η. The asset prices are $Q(\eta), \theta(\eta)$, and the flow allocation is the fraction of capital held by banks $\psi(\eta)$; we suppress time subscripts to reduce notation unless necessary for clarity. We derive the evolution of η_t using Ito's Lemma and the equations for returns and budget constraints.

Lemma 1. *The equilibrium law of motion of η_t will be endogenously given as*

$$\frac{d\eta_t}{\eta_t} = \mu_{\eta,t}\,dt + \sigma_{\eta,t}\,dW_t + d\Xi_t, \tag{14}$$

where $d\Xi_t$ is an impulse variable creating a regulated diffusion. Furthermore,

$$\mu_{\eta,t} = \left(\frac{\psi_t}{\eta_t} - 1\right)\big((\sigma_{ch,t} - \sigma_{\theta,t}) - (\sigma + \sigma_{Q,t})\big)(\sigma + \sigma_{Q,t}) + \frac{1}{Q_t} - \mathscr{T}_{b,t} + (1-\psi_t)(g_b - g_h),$$
$$\sigma_{\eta,t} = \frac{(\psi_t - \eta_t)}{\eta_t}(\sigma + \sigma_{Q,t}), \quad d\Xi_t = \frac{d\zeta_t}{N_{b,t}},$$

where $d\zeta_t = \int d\zeta_{b,t}db$ and $\psi_t = \frac{1}{Y_t}\int x_{b,t}n_{b,t}db$ is the fraction of the tree held by banks.

Equilibrium Consumption and Asset pricing Output in equilibrium is the tree dividend net of intermediation costs ($\iota \equiv \kappa m q^D$) from digital currency. Aggregate consumption is given by

$$C_t = (1-\iota)Y_t,$$

which means that consumption growth is $\mu_{ch,t} = g_{y,t}$ and consumption volatility is $\sigma_{ch,t} = \sigma$.[6] As a result, we can write the risk-free rate as (see Lemma 2)

$$r_{f,t} = r + g_{y,t} - \sigma^2.$$

[6] Constant consumption volatility makes the model computationally tractable and is why we consider a constant ι, i.e., a fixed m.

Households hold wealth in capital, bank equity, and bank deposits, which in the aggregate are $(1-\psi)QY$, θN_b, and $\psi QY - N_b$. Households hold government liabilities, which equal the present value of future taxes to pay the returns on those assets together with future issuance. However, the present value of future taxes also includes the cost of intermediating digital currency, which is a flow cost of ιY_t.[7] Discounted using households' SDF, the present value of this flow per unit of Y is

$$\frac{\iota}{r_f - g + \sigma^2} = \frac{\iota}{r}.$$

Thus, total household wealth satisfies

$$W_h = Q(1+(\theta-1)\eta)Y - \frac{\iota}{r}Y = \left(Q(1+(\theta-1)\eta) - \frac{\iota}{r}\right)Y,$$

Market clearing for consumption is $rW_h = (1-\iota)Y$, and so $W_h = \frac{1-\iota}{r}Y$, hence

$$Q(\eta) = \frac{1}{r(1+(\theta(\eta)-1)\eta)}. \tag{15}$$

The aggregate value of deposits and digital currency as a fraction of Y_t are

$$Depo_t = (\psi_t - \eta_t)Q_t, \quad \mathbb{DC}_t = mq^D, \quad L_t = \left(((\psi_t - \eta_t)Q_t)^{\frac{\varepsilon-1}{\varepsilon}} + \delta\left(mq^D\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}},$$

In equilibrium, the shares of household wealth held in deposits and digital currency are

$$x_{d,t} = \frac{r(\psi_t - \eta_t)Q_t}{1-\iota}, \quad x_{dc,t} = \frac{rmq^D}{1-\iota}, \quad \ell_t = \frac{r}{1-\iota}L_t,$$

and the spreads on deposits and digital currency are

$$DS_t = r\frac{\beta}{\ell_t}\left(\frac{\ell_t}{x_{d,t}}\right)^{\frac{1}{\varepsilon}}, \quad DCS_t = r\frac{\beta}{\ell_t}\delta\left(\frac{\ell_t}{x_{dc,t}}\right)^{\frac{1}{\varepsilon}}.$$

[7] In the case of private provision of digital currency, this would be included in transfers to firms.

Note that spreads are decreasing in liquidity supply ℓ generally, and each spread decreases in the specific liability, i.e., DCS_t decreases with $x_{dc,t}$ and therefore with m.

To characterize the equilibrium asset pricing conditions, we first define households' required risk premium and banks' solvency risk premium as

$$HP_t \equiv \sigma(\sigma + \sigma_{Q,t}), \quad SP_t \equiv -\sigma_{\theta,t}(\sigma + \sigma_{Q,t}).$$

The solvency risk premium SP_t represents the portion of banks' required risk premium (or instantaneous level of risk aversion) caused by costly equity issuance and solvency concerns. Household investment in capital implies

$$\mathbb{E}[dr_{h,t}] - r_{f,t} \leq HP_t, \tag{16}$$

which holds with equality whenever households invest in the tree. Banks' portfolio choice implies

$$\mathbb{E}[dr_{b,t}] - r_{f,t} = HP_t + SP_t - DS_t. \tag{17}$$

Relative to households, banks earn extra returns of at least $(g_b - g_h) + DS_t - SP_t$: they have a productivity advantage $g_b - g_h$ and a funding advantage DS_t but bear the additional risk SP_t due to financial frictions. Differencing (17) and (16) yields the equilibrium asset pricing condition:

$$g_b - g_h + DS_t \geq SP_t, \tag{18}$$

which binds whenever households hold a positive quantity of the tree.

Solving the ODE While there are two dynamic valuation variables (Q and θ), it is sufficient to solve for the dynamics of θ as a function of η since Q is a function of θ (equation (15)). We solve for equilibrium by converting the equilibrium conditions into a system of differential equations ("ODE") in θ. Given $\theta(\eta), \theta'(\eta)$, we can get equilibrium returns and allocations to get $\theta''(\eta)$. Equation (15) allows us to solve for $Q(\eta)$ and $Q'(\eta)$. We solve the ODE using appropriate boundary conditions.

Proposition 3 (Equilibrium)**.** *The equilibrium domain of the functions $Q(\eta), \theta(\eta)$, and $\psi(\eta)$ is an interval $[\underline{\eta}, \overline{\eta}]$. The function $Q(\eta)$ is increasing, $\theta(\eta)$ is decreasing, and the following boundary conditions hold: (i) $\theta(\overline{\eta}) = 1$; (ii) $\theta(\underline{\eta}) = 1+\gamma$; (iii) $\theta'(\overline{\eta})\overline{\eta} + \theta(\overline{\eta}) = 1$; (iv) $\theta'(\underline{\eta})\underline{\eta} + \theta(\underline{\eta}) = 1$. Over $[\underline{\eta}, \overline{\eta}]$, $\theta_t \geq 1$ and $d\zeta_t = 0$. At $\overline{\eta}$, $d\zeta_t > 0$, and at $\underline{\eta}$, $d\zeta_t < 0$, creating regulated barriers for the process η_t.*

The first two boundary conditions reflect optimal policy for dividends/equity issuance. The last two conditions are smooth pasting conditions implying $Q'(\overline{\eta}) = 0$ and $Q'(\underline{\eta}) = 0$. Note that combining the conditions at $\overline{\eta}$ implies that $\theta'(\overline{\eta}) = 0$. For high levels of η, banks can hold the entire capital stock. The evolution of η induces a stationary density (PDF) $f(\eta)$ with CDF $F(\eta)$; the density $f(\eta)$ solves a Kolmogorov-Forward equation.

If the price function and marginal value of bank equity are twice-continuously differentiable, then equations (2) and (13) are functions of η:

$$\frac{dQ_t}{Q_t} = \mu_Q(\eta_t)\,dt + \sigma_Q(\eta_t)\,dW_t, \quad \frac{d\theta_t}{\theta_t} = \mu_\theta(\eta_t)\,dt + \sigma_\theta(\eta_t)\,dW_t, \tag{19}$$

where the drift and variance terms are determined by the derivatives of $Q(\eta)$ and $\theta(\eta)$. (For the remainder of the paper, the dependence on the state-variable η_t is suppressed for notational ease.)

Financial Stability Financial frictions generate systemic risk because outcomes depend on the evolution of bank equity η_t. Accordingly, the behavior of η_t determines the stability of the system (i.e., financial stability). The fundamental problem caused by financial frictions is the inability of banks to always hold the entire capital stock and issue deposits. Households are worse-off when fire sales are frequent and deep because output growth and liquidity services fall during fire sales. Our measures of stability directly quantify this problem. There are a number of different variables that capture financial stability in the model.

To start, it is useful to divide the state space into three regions and label them. We define a *boom* as when banks hold the entire capital stock. The economy is in good times during a boom. We define *distress* as when banks are so constrained that they have to sell capital at fire-sale prices to households (i.e., $\psi < 100\%$). We define a *crisis* as when households hold at least 50% of the capital stock ($\psi < 50\%$), indicating severe misallocation due to fire sales. Thus, the economy is in bad times when it is in distress or a crisis. The stability of the economy can be measured by the long-run probability the economy is not in distress or the long-run probability the economy is not in crisis.[8]

Additional measures of financial stability also include μ_η and σ_η. The drift μ_η captures the speed with which banks recover equity following losses (higher is more stable), and the systemic volatility σ_η captures how much the financial sector amplifies fundamental shocks (lower is more stable). Together these terms determine the stationary distribution $f(\eta)$ through the Kolmogorov-Forward equation, which determines the probabilities of crises and distress. Finally, asset price volatility σ_Q is a measure of endogenous risk in financial markets that is related to but distinct from banking-sector stability per se.

Private Issuance of Digital Currency We solve the model for a given supply m of digital currency without concern for what entity issued it or whether it is profitable to do so. Nonetheless, it is constructive to consider what supply of digital currency would occur in equilibrium in a competitive market of privately issued digital currency. To maintain tractability, we suppose that potential digital currency issuers make a once-and-for-all decision to issue μ units of currency per unit of Y_t, automatically scaling issuance with output. We then determine the level of digital currency consistent with equilibrium.

Let J denote the value of issuing digital currency. The flow payoff to issuing currency is $(DCS(\eta) - \kappa)\mu Y$. Discounted by the household SDF, we divide by consumption, which yields a discounted flow payoff of $z(\eta) \equiv (DCS(\eta) - \kappa)\frac{\mu}{1-\iota}$, where $\frac{\mu}{1-\iota}$ is a constant and Y drops out. The

[8] He and Krishnamurthy (2019) use a similar measure of stability. They define the threshold for financial distress as the equity level at the 33rd percentile based on the stationary distribution, and they define a crisis as when a capital constraint binds for financial intermediaries. In our model, the equivalent notion of a binding capital constraint is precisely the sale of capital by banks because low bank equity restricts their ability to hold capital. However, because the output losses are relatively modest when households hold a small share of capital, we choose to reserve the label of "crisis" for when households hold at least 50% of the capital stock.

value J therefore satisfies the HJB

$$rJ(\eta) = z(\eta) + (\eta\mu_\eta)J_\eta + \frac{1}{2}(\eta\sigma_\eta)^2 J_{\eta\eta}.$$

The current value J clearly depends on the current state η. We therefore make a second assumption that potential issuers do not know η precisely but make an ex-ante decision based on a "timeless" perspective on the initial condition η: the initial condition is not known, but they take expectations according to the stationary distribution. The stationary density $f(\eta)$ determines the ex-ante distribution of initial conditions (capitalization of the financial sector) and agents then compute $\mathbb{E}[J(\eta)]$ using the stationary distribution occurring in equilibrium. Thus, issuers will enter whenever $\mathbb{E}[J(\eta)] > 0$. Recall that DCS_t is decreasing in m and so when $\mathbb{E}[J(\eta)] > 0$ digital currency issuance will increase, driving down spreads. Thus, we say that the competitive equilibrium level of digital currency is the $\hat{m}$ that satisfies $\mathbb{E}[J(\eta)] = 0$.

2.4 Welfare and Efficiency

Because banks are owned by households and are not competing agents, the model aggregates up to a representative household. Aggregate welfare can be written

$$V_t = \mathbb{E}_\tau \left[\int_t^\infty e^{-r(\tau-t)} \left(\log((1-\iota)Y_\tau) + \beta \log(\ell_\tau) \right) d\tau \right], \tag{20}$$

where consumption is $C_t = (1-\iota)Y_t$. Because households have log utility, we can write their value function as

$$V_t = \frac{\log(W_{h,t})}{r} + H_t,$$

where H_t is a wealth-independent term. Because total household wealth satisfies $W_h = \frac{1-\iota}{r}Y$, the value function can be re-written as a function of η and Y:

$$V(\eta, Y) = \frac{\log(Y)}{r} + H(\eta) + \frac{A}{r}, \tag{21}$$

where A is a constant defined by parameters. The function $V(\eta, Y)$ satisfies the HJB

$$\begin{aligned} rV(\eta, Y) = &\log(1-\iota) + \log(Y) + \beta \log(\ell(\eta)) \\ &+ (\eta\mu_\eta)V_\eta + \frac{(\sigma_\eta \eta)^2}{2} V_{\eta\eta} + g_y Y V_Y + \frac{(Y\sigma)^2}{2} V_{YY}. \end{aligned}$$

From (21), $V_Y = \frac{1}{rY}$ and $V_{YY} = -\frac{1}{rY^2}$. Plugging in and collecting terms, this simplifies to

$$\begin{aligned} \log(Y) + rH(\eta) + A = &\log(1-\iota) + \log(Y) + \beta \log(\ell(\eta)) \\ &+ (\eta\mu_\eta)V_\eta + \frac{(\sigma_\eta \eta)^2}{2} V_{\eta\eta} + \frac{1}{r} g_y(\eta) - \frac{\sigma^2}{2r}, \end{aligned}$$

and hence

$$\begin{aligned} rH(\eta) &= \beta \log(\ell(\eta)) + \frac{1}{r} g_y(\eta) + H'(\eta)\eta\mu_\eta + \frac{(\sigma_\eta \eta)^2}{2} H''(\eta), \\ A &= \log(1-\iota) - \frac{\sigma^2}{2r}. \end{aligned} \tag{22}$$

Equation (22) clarifies the source of welfare variations in the economy. Welfare varies with η because the growth rate of capital depends on banks' asset holdings and because liquidity services depend on banks' provision of deposits. The function $H(\eta)$ is therefore the expected discounted value of the convenience yield from liquidity and the economy growth rate. We can translate welfare consequences into consumption-equivalent values. Consider allocations 0 and 1 that yield welfare values V_0 and V_1. Then the consumption-equivalent of moving from allocation 0 to 1 is

$$CE = e^{r(V_1 - V_0)} - 1.$$

Losses incorporate the decreased growth consequences from capital misallocation (when $\psi < 1$) and also the consequences of liquidity underprovision.

First-Best It is instructive to consider how financial frictions affect welfare in equilibrium, and how digital currency operates in the first-best. It is easy to characterize the first-best equilibrium.

When banks can freely issue equity, then $\theta = 1$. Furthermore, banks fund themselves entirely with deposits because there is no risk of bankruptcy, and banks hold the entirety of the tree. With financial frictions, banks use equity as a buffer against bankruptcy, which decreases liquidity provision.

Proposition 4 (First-Best)**.** *In the absence of financial frictions (free equity issuance), the first-best equilibrium satisfies $\psi_{FB} = 1$ and $Q_{FB} = \frac{1}{r}$. Furthermore welfare satisfies*

$$rV_{FB} = \log(1-\iota) + \beta \log(\ell_{FB}) + \frac{g_b}{r} - \frac{\sigma^2}{2r} + \log(Y),$$

where ℓ_{FB} is the CES aggregator over $x_{d,FB} = \frac{rQ_{FB}}{1-\iota} = \frac{1}{1-\iota}$ and $x_{dc,FB} = \frac{rmq^D}{1-\iota}$.

Welfare in the first-best depends on the level of digital currency. Higher m decreases output via $\iota \equiv m\kappa q^D$. The effect on liquidity is more subtle: increasing m increases digital currency directly but also increases the fraction of wealth held in deposits because overall wealth *decreases*. Denote the optimal level of digital currency in the first-best by m^*_{FB}.

Proposition 5. *In the first-best, a positive level digital currency is optimal ($m^*_{FB} > 0$) whenever $\varepsilon < \infty$. For an interior solution, m^*_{FB} satisfies*

$$\kappa(1-\beta) = DCS_{FB}(m^*_{FB}),$$

where $DCS_{FB}(m^*_{FB})$ is the digital currency spread in the first-best equilibrium with supply m^*_{FB} of digital currency. Hence $m^*_{FB} > \hat{m}_{FB}$: the socially-optimal level exceeds what would be issued by private competitive markets.

The planner internalizes that the fraction of wealth held in deposits affects the equilibrium/aggregate level of liquidity services. The planner optimally issues more DC at the expense of lower output because it also decreases household wealth (via ι), which raises liquidity services As a result, the socially-optimal level of digital currency exceeds what would be produced privately ($\hat{m}_{FB}$ satisfies $\kappa = DCS(\hat{m}_{FB})$). Additionally, if $\varepsilon < \infty$, then $m = 0$ is never optimal because the digital currency spread at $m = 0$ is infinite (recall $\frac{\partial \ell}{\partial x_{dc}} = \delta \left(\frac{\ell}{x_{dc}}\right)^{\frac{1}{\varepsilon}}$, which is infinite if $x_{dc} = 0$). For κ sufficiently low, the optimal could be $m^*_{FB} = 1$ (certainly the case when $\kappa = 0$).

3 Calibration

This section discusses the benchmark calibration for the model parameters. We calibrate the model in the absence of digital currency ($m = 0$). Our quantitative results then consider the consequences of introducing digital currency for financial stability and welfare.

Our calibration strategy intentionally gives banks a critical role in the allocation of capital. This decision actively reflects the concerns among economists and policymakers that digital currency have the potential to lead to significant disintermediation of the banking sector, resulting in inefficient allocation of capital. Disintermediation matters to the extent that banks are important for capital allocation. In this calibration, output growth suffers significantly when households hold the tree (i.e., when there is less intermediation) and the expected output losses from crises are large.

Nonetheless, there are good arguments to make for an alternative calibration that does not a priori give banks such a prominent investment role. Appendix C presents an alternative calibration strategy in which banks do not have a productivity advantage ($g_b = g_h$) and deposit spreads are much lower, corresponding more to the convenience yields seen in short-term funding markets rather than bank deposits. Perhaps surprisingly, even though in that calibration banks are much less important for liquidity provision and are not at all important for the capital allocation, the overall results of our paper continue to hold in that very different calibration.

Parameters Table 1 reports our calibration for the parameters in the baseline model and Table 2 reports the empirical moments we target. Several parameters are disciplined by the existing empirical literature, while the remaining parameters are calibrated by jointly targeting empirical moments from U.S. data. We target 5 empirical moments, which are common targets in the literature: probability of crises, average bank leverage, average deposit spreads, average Sharpe ratio for banks, and average output loss from a crisis. The remaining parameters do not affect equilibrium when there is no digital currency, so we leave their discussion until later.

A discount rate of $r = 2\%$ and an annualized growth rate of $g_b = 2\%$ are standard values in the literature (see Drechsler et al., 2018; He and Krishnamurthy, 2019). Average GDP growth volatility is 2.2% over the period 1950-2019. We use a slightly higher value in order to hit the

Table 1: Baseline model parameters, calibrated according to the existing literature or by matching empirical moments for U.S. data.

Description	**Parameter**	**Value**
Discount Rate	r	2%
Typical Growth Rate	g_b	2%
Fundamental Volatility	σ	2.25%
Liquidity Preference	β	0.3672
Household Growth	g_h	0.3%
Equity Issuance Cost	γ	100
Bank Transfers	$\mathscr{T}_b$	14%

moments in the data; our choice of 2.25% is similar to values used in the literature (Drechsler et al. (2018) calibrate to 2% and He and Krishnamurthy (2019) use 3%).

We calibrate β to match average deposit spreads. Drechsler et al. (2017) calculate average deposit spreads of 108 basis points. Begenau and Landvoigt (2021) calculate a market leverage ratio of 10 for banks over the period 1999-2019. Following He and Krishnamurthy (2019), we target a 3% long-run probability of crisis, which we define as whenever $\psi < 50\%$ (Chen and Phelan, 2022b). He, Kelly, and Manela (2017) estimate an average Sharpe ratio of 48%, and Jordà, Schularick, and Taylor (2013) estimate average output losses of 8% in crises. Our average Sharpe ratio is close to He et al. (2017)'s empirical estimate as well as He and Krishnamurthy (2019)'s calibration, which yields a Sharpe of 45%. To calculate expected output losses in a crisis, we simulate our model over five years starting from a crisis (when $\psi = 50\%$) and compare total output to what would have occurred if the economy had remained at the stochastic steady state.

It's worth understanding how the remaining parameters (g_h, γ, and $\mathscr{T}_b$) help us match the empirical targets. In the absence of financial frictions, banks would hold the entire tree, and leverage and average returns would be determined by fundamental risk σ and banks' funding advantage determined by the deposit spread. Generating empirically realistic output losses in a crisis requires a relatively large growth discount when household however the tree. However, a low g_h also implies large excess returns for banks when households hold the asset. This creates two challenges for the calibration. First, high returns provide incentives for banks to hold the asset rather than sell to

Table 2: Targeted empirical moments. Averages of variables are computed by integrating the moment of interest with respect to the stationary density.

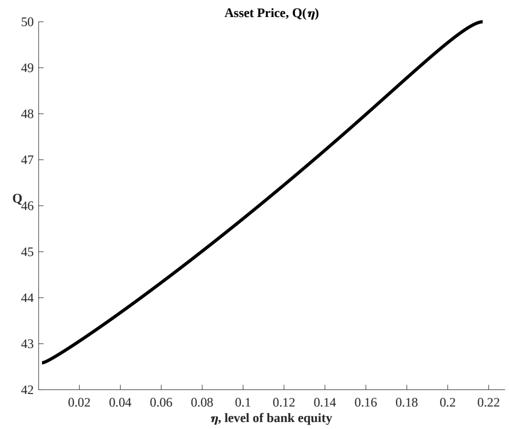
Empirical Moment	**Data**	**Model**
Average Deposit Rate	108 bps	108.6 bps
Average Bank Leverage	10	10.11
Prob. of Crisis ($\psi < 50\%$)	3%	2.99%
Average Sharpe Ratio	48%	47.26%
Output Loss in Crisis	8%	8.04%

households, which limits fire sales. Thus, generating crises requires significant financial frictions (high issuance costs), otherwise following losses banks will simply hold assets and issue equity (generating crises requires the reverse). Second, high returns imply that banks rebuild equity very quickly, so the system is very stable. Achieving high average levels of leverage and high average Sharpe ratios require large bank transfers $\mathscr{T}_b$.[9]

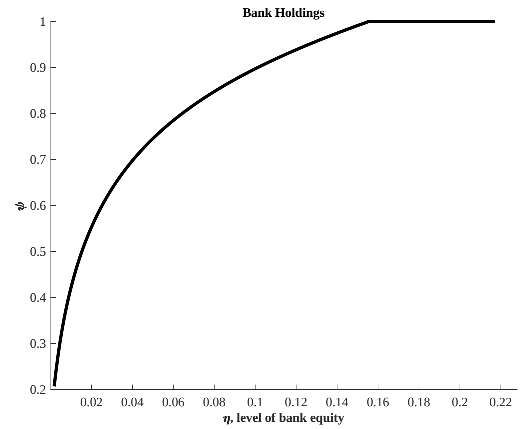
The model does a good job of matching the aggregate empirical moments, but the calibrated values of issuance costs and bank transfers strikes us as implausibly large. Nonetheless, the model is somewhat stylized and so these parameters are likely capturing unmodeled features.

Model Dynamics Figure 1 displays equilibrium prices, allocations, and dynamics for the calibrated model. When banks are well-capitalized, they own the entire capital stock, and asset prices are high. In this region, asset price volatility is nearly zero, and fundamental risk explains the majority of the volatility in η. As η decreases and banks become unwilling to hold the entire capital stock, fire sales depress asset prices, and volatility spikes. The kink in each plot occurs at $\overline{\eta}$, the threshold for fire sales. Bank leverage is a decreasing function of η: losses induce asset sales, but because banks use leverage any losses from their assets have multiplied consequences for their

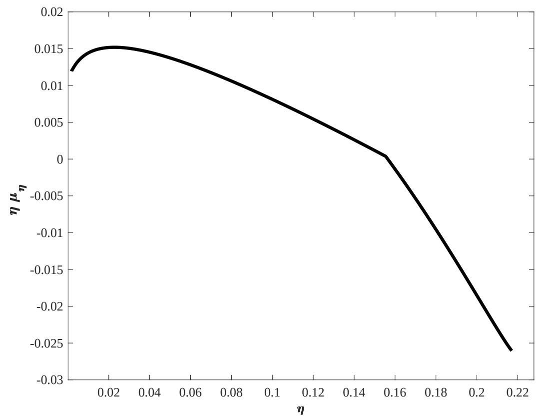
[9] These outcomes are examples of the *intermediation paradox* pointed out by Phelan (2016). Although productivity losses create adverse feedback between fire sales and output in bad times, the economy is counterintuitively more stable due to general equilibrium effects. Large productivity losses are stabilizing because in equilibrium they raise the expected returns earned by banks, which help banks recapitalize faster, promoting long-run stability. Without this endogenous "hedge" when bank equity is low, banks would deleverage through fire sales sooner, making financial crises more likely. All else equal, fire sales become less probable in equilibrium when g_h is lower.



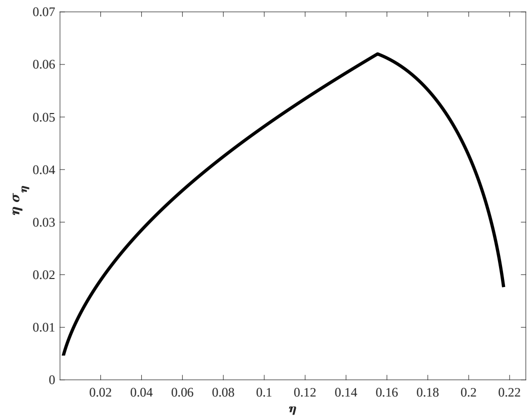
(a) Asset Price $Q(\eta)$



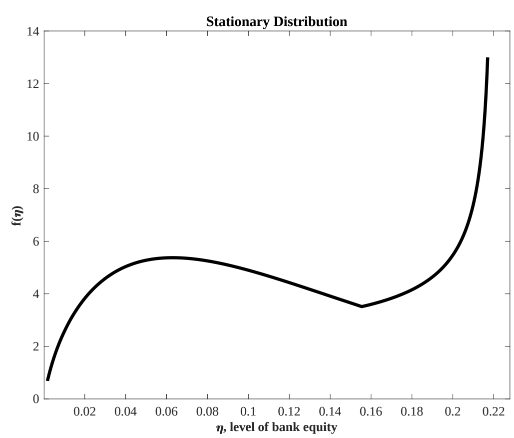
(b) Bank holdings $\psi(\eta)$



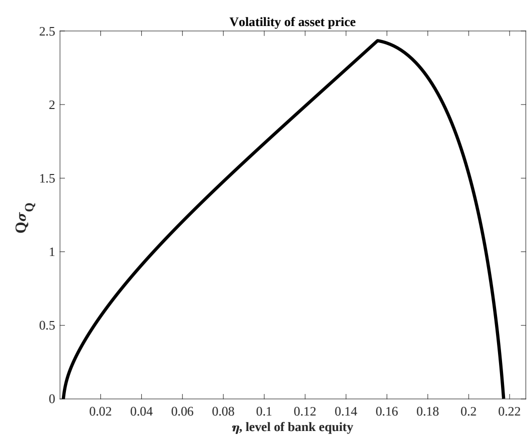
(c) Systemic drift $\eta\mu_\eta$



(d) Systemic volatility $\eta\sigma_\eta$



(e) Stationary Density $f(\eta)$

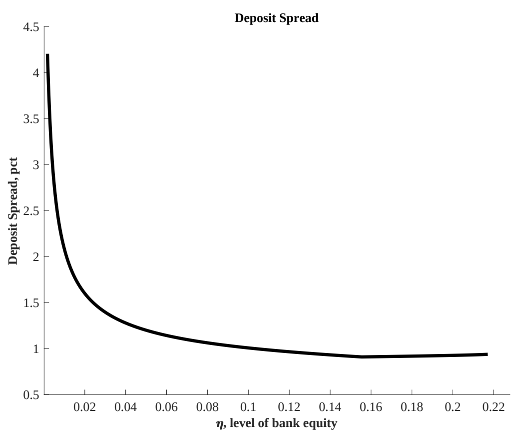


(f) Endogenous price volatility $Q\sigma_Q$

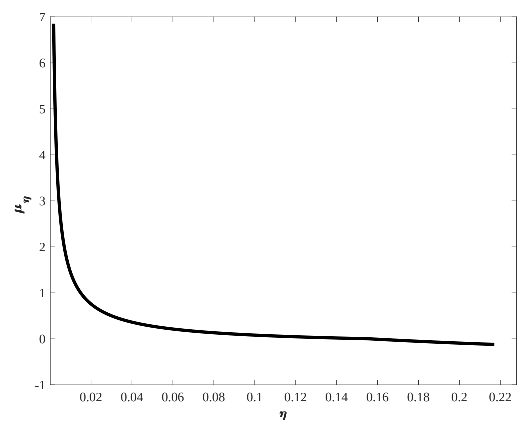
Figure 1: Equilibrium prices, allocations, evolutions, and stationary density in baseline economy. In each panel, the horizontal axis is the state variable η, which is the ratio of banks' book equity to the total value of capital.

equity, which is why leverage rises following losses. When η approaches zero, the economy is less volatile because banks are too small to have sizable aggregate impacts.

In the model, deposit spreads move mechanically with total debt and do not reflect potential concerns about bank solvency (all bank deposits are risk-free). In times of crisis, bank deposit *rates* decline dramatically because deposit *spreads* increase (because deposit issuance is low). While this prediction about spreads is counterfactual, it captures the important real-world dynamic that deposits tend to flow *into* banks during crises, providing banks with a stable funding source during crises. In the same way, the increase in deposit spreads in crises provides an important stabilizing force for banks. Figure 2 illustrates. Panel (a) plots deposit spreads and panel (b) plots the percent system drift μ_η (recall that the system drift plotted above is $\eta\mu_\eta$). For low η, funding markets provide an automatically stabilizing force because banks can more cheaply fund their balance sheet, which allows them to recapitalize more quickly (hence a high μ_η).



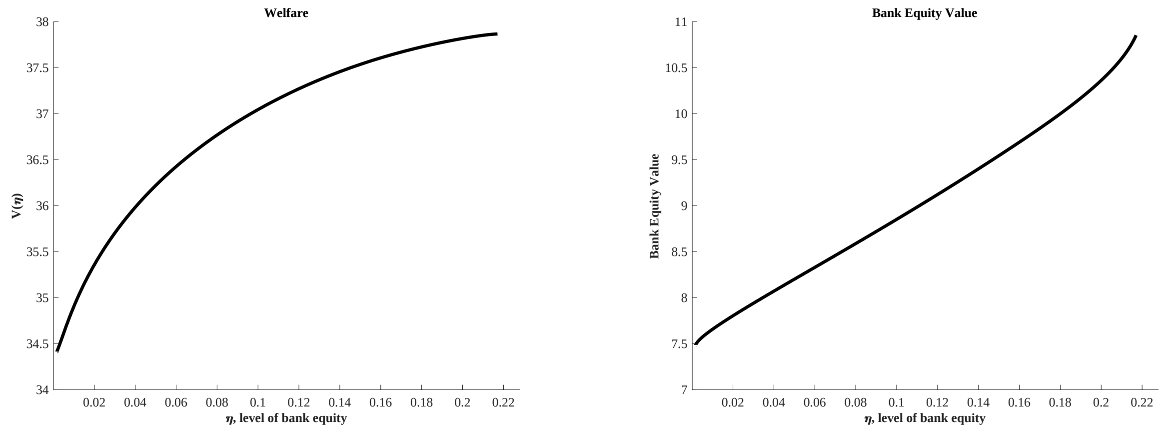
(a) Deposit Spreads



(b) System Drift μ_η

Figure 2: Deposit spreads and financial stability.

Welfare and Bank Values Household welfare depends on the initial condition η, as illustrated in Figure 3. Thus, the welfare consequence of digital currency may vary with the initial condition. As we did with private issuance of digital currency, in our later analysis we will take a "timeless" perspective on the initial condition η: the initial condition is not known, but we take compute $\mathbb{E}[V(\eta)]$ using the stationary distribution occurring in equilibrium.



(a) Household Welfare (b) Bank Valuations

Figure 3: Welfare and bank valuations in the baseline model

Figure 3 also plots the aggregate value of bank equity, which also varies with η. While households do not care about bank equity per se—welfare depends on total wealth, not necessarily the composition of wealth, and households ultimately derive utility from consumption and liquidity flows, not bank valuations—the effects of digital currency on bank valuations are an important variable to note for reasons of political economy. As we show below, digital currency can substantially improve household welfare while having very negative consequences for bank valuations. This creates misaligned incentives between the banks and the households who own the banks. The financial sector could rationally oppose digital currencies while digital currencies could be welfare-improving. As we do for welfare, going forward we will consider the average bank valuation.

Digital Currency Parameters There are four remaining parameters that we need to calibrate to determine the consequences of digital currency issuance in equilibrium. Table 3 lists the benchmark values used for parameters that determine the consequences of adding digital currency. We calibrate government debt by targeting federal debt as a fraction of total liquidity provision. This parameter determines the potential supply of digital currency that can be issued. In the model, private debt per unit of Y is $Q(\psi-\eta)$, which we proxy using M2, which includes bank deposits and money market funds. The ratio of government debt to M2 is 1.4. The baseline model with $m=0$ produces an average of 1.4.

Table 3: Model parameters determining liquidity value of digital currency.

Description	**Parameter**	**Value**
Government debt supply	q^D	47.5
Elasticity of substitution	ε	5.3
Currency preference	δ	1
Intermediation cost	κ	100bps

There are three key parameters related to digital currency that are important for our results but which we are uncomfortable definitively determining. The first two parameters determine the preference for digital currency relative to bank deposits: the elasticity of substitution ε and the relative preference δ. One sensible benchmark for ε is the elasticity between cash and deposits, which Drechsler et al. (2017) estimate as 5.3. We set this as our benchmark but also consider a higher elasticity of 53 as well. The results with $\varepsilon = 53$ very closely approximate the case when DC and deposits are perfect substitutes (i.e., $\varepsilon = \infty$). Since the (perhaps yet to be determined) particular technologies for CBDC and stablecoins determine the parameter δ, we set $\delta = 1$ as a prior (liquidity comparable to deposits) and do robustness analysis.[10]

The third parameter of interest is the cost κ of issuing a digital currency. This cost is important for the normative questions of welfare. As a benchmark, we set $\kappa = 100$bps so that the cost is roughly equal to the average deposit spread (we target 108bps for average deposit spreads).

In the results that follow, we consider the robustness of our results by varying these three parameters.

4 Quantitative Results: Stability and Welfare

Our main quantitative results concern how the supply of digital currency—a source of liquidity services that competes with bank deposits—affects financial stability and household welfare. Appendix C presents an alternative calibration strategy with lower deposit spreads and in which banks

[10] As already noted, empirical estimates suggest that digital currencies are not presently good money (Gorton et al., 2022) but instead carry an *inconvenience yield* at present. Risks to so-called stablecoins could be captured by low δ.

do not enjoy a productivity advantage. While this calibration yields very different parameter values, it nonetheless produces the same qualitative results.

4.1 Benchmark Results

We first consider the quantitative consequences of increasing digital currency to $m = 18.27\%$ and $m = 26.1\%$.[11] As we show in Appendix B, the equilibrium consequences of converting a fraction $m \in (0,1)$ are qualitatively the same, and in many cases the *quantitative* significance scales almost linearly with m. Table 4 provides the behavior of key model moments as a result of digital currency issuance. As expected, deposit spreads decrease with digital currency, which provides a competing form of liquidity. Figure 5 in Appendix B plots the equilibrium variables when increasing digital currency to $m = 18.27\%$ and $m = 26.1\%$.

Table 4: Model outcomes with baseline calibration, $\varepsilon = 5.3$. Averages of variables are computed by integrating the moment of interest with respect to the stationary density.

Moment	**Baseline** $m = 0$	$m = 18.27\%$	$m = 26.1\%$
Average Deposit Rate	108.6 bps	76.9 bps	68.7 bps
Average Bank Leverage	10.11	11.34	11.79
Prob. of Crisis ($\psi < 50\%$)	2.99%	5.97%	7.20%
Prob. of Distress ($\psi < 100\%$)	65.77%	70.49%	71.91%
Average Sharpe Ratio	47.26%	47.43%	47.61%
Output Loss in Crisis	-8.06%	-8.78%	-8.99%
Average Bank Holdings ψ	87.03%	83.41%	82.19%
Average Bank Equity η	11.52%	10.24%	9.86%
Asset Price Volatility σ_Q	3.19%	2.77%	2.64%
System Drift $\eta\mu_\eta$	0.20%	0.18%	0.17%
System Volatility $\eta\sigma_\eta$	4.20%	3.79%	3.67%
Average Asset Price Q	46.37	46.38	46.40
Welfare Gains (CE)	–	2.81%	2.47%
Change in Bank Valuations (pct)	–	-6.80%	-8.94%

Several robust results emerge. First, digital currencies harm the stability of the financial sector.

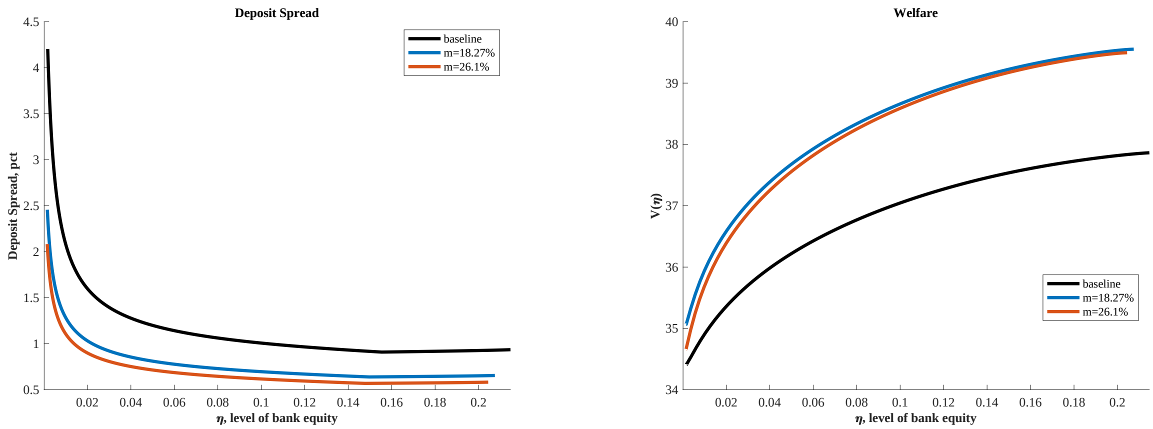
[11] These choices of m correspond to the optimal levels in the model with frictions and in the first-best setting, discussed in greater detail below.

Digital currency increases the probability of crises and the probability of distress. The financial system rebuilds equity more slowly following losses (the system drift is lower) resulting in lower average levels of equity. This last result should be surprising. Digital currency makes deposit funding less attractive (lower spreads). In a static environment, that would make equity financing relatively more attractive for banks, which would suggest that, all else equal, banks would shift toward *more* equity financing because deposits are less attractive. Precisely the opposite happens on average with financial frictions. Because banks are less able to rebuild equity after adverse shocks (recall that equity *issuance* is expensive), banks on average have lower equity. Accordingly, bank valuations decrease significantly.

Second, these levels of digital currency issuance improve household welfare, and the quantitative sizes are significant, with welfare gains of more than 2% consumption-equivalents. Indeed, there are some measures of stability that improve. In particular, the volatility of the system declines, which primarily reflects a decrease in the volatility of asset prices. Average asset prices increase with the issuance of digital currency, which is significant precisely because stability has worsened (the asset price is lower when η is lower). Average asset prices increase because the function $Q(\eta)$ increases for all η even though the distribution $f(\eta)$ worsens. Thus, financial *markets* improve, with lower volatility and higher prices, but the financial *sector* suffers.

Figure 4 plots the effects of digital currency on deposit spreads and household welfare over the state space. Digital currency pushes down deposit spreads in general, but especially in times of crisis when banks most desperately need to rebuild equity. The decline in deposit spreads for low η is the primary reason that the probability of crisis increases with digital currency issuance. Nonetheless, welfare is significantly improved by these levels of digital currency for every value of η.

The positive results presented here are robust along a number of dimensions. First, the stability consequences of digital currency are monotonic in m: higher levels of m further decrease financial stability. Second, the results are robust to considering a higher elasticity between deposits and digital currency, varying costs of issuing currency, and varying the preference δ for digital currency. We provide these results in Appendix B. The normative welfare results are somewhat



(a) Deposit Spreads

(b) Welfare

Figure 4: The consequences of digital currency for deposit spreads and Welfare.

more subtle. Welfare is generally hump-shaped in m, as is also true in the first-best (Proposition 4). Accordingly, the optimal level of digital currency is parameter-dependent.

4.2 Welfare and Optimal Level of Digital Currency

We now consider the welfare consequences and optimal level of digital currency in greater detail. Proposition 4 showed that in the absence of financial frictions, there was an aggregate externality so that the optimal level of digital currency exceeded what would be provided in a competitive market. We show that financial frictions can mitigate or even reverse this result.

Table 5 provides the optimal and equilibrium supply of digital currency with and without financial frictions, varying the elasticity of substitution ε and the preference for digital currency δ. The equilibrium levels $\hat{m}$ denote the supply of digital currency that would be produced in a competitive environment by private issuance. The optimal levels are denoted by m^*. The subscript FB denotes the first-best and the subscript 1 denotes the model with frictions. Recall that the case with $\varepsilon = 53$ closely approximates the case when DC and deposits are perfect substitutes.

As we found in our analytical result Proposition 4, it is always the case that $m^*_{FB} > \hat{m}_{FB}$. It is not too surprising that $\hat{m}_{FB} < \hat{m}_1$: in the first-best banks fund themselves entirely with deposits (no equity), and therefore deposit spreads are lower than occur in the model with financial frictions.

Table 5: Optimal and equilibrium supply of digital currency with and without financial frictions. This table provides the competitive levels $\hat{m}$ together with the optimal levels m^*. The subscript FB denotes the first-best and the subscript 1 denotes the model with frictions.

	First-Best		*Frictions*	
Currency preference	$\hat{m}_{FB}$	m^*_{FB}	$\hat{m}_1$	m^*_1
		Baseline elasticity $\varepsilon = 5.3$		
$\delta = 1.25$	9.0%	22.1%	14.7%	14.3%
$\delta = 1.00$	9.9%	26.1%	16.8%	17.3%
$\delta = 0.50$	12.9%	42.4%	24.6%	26.9%
$\delta = 0.25$	15.6%	65.6%	30.8%	39.3%
		Higher elasticity $\varepsilon = 53$		
$\delta = 1.00$	≈ 0%	13.3%	6.4%	1.4%
$\delta = 0.50$	≈ 0%	25.0%	9.2%	1.4%
$\delta = 0.25$	≈ 0%	46.8%	7.8%	4.3%

Thus, with higher deposit spreads, there is a greater incentive to issue digital currency in the model with frictions. What is most notable is the relationship of m^*_1 with $\hat{m}_1$ and m^*_{FB}.

First, and most importantly, financial frictions always decrease the optimal level of digital currency relative to what would occur in the first-best. In the first-best there are no concerns about financial stability; with financial frictions, digital currency harms financial stability, thus decreasing the social benefit of digital currency.

Second, it is possible for the optimal level of digital currency to be *less* than what would be privately produced in competitive equilibrium. This is precisely the opposite of what happens in the absence of frictions. We see this in each case with high elasticity and in the baseline elasticity when $\delta = 1.25$. In these cases, the negative aggregate externalities coming from financial instability outweigh the positive aggregate externality that is virtually hard-coded into the model.

As we have said, we do not take a stand on what "the right" values of ε and δ should be. But it strikes us that most of the existing literature assumes that digital currencies, and especially

CBDC, would be highly substitutable with bank deposits, and several authors point out reasons that CBDC would dominate deposits (Fernández-Villaverde et al., 2021). If that's the case, then our quantitative results suggest that—taking as given the current environment in which banks face financial frictions—optimal regulation would limit the supply of digital currencies.

4.3 Discussion

Calibration Robustness In our benchmark calibration we considered the financial sector as primarily corresponding to the banking sector: deposit spreads are high, and output losses from disintermediation are significant. There may be good reasons to question those assumptions specifically and to wonder about the robustness of our results to alternative calibrations more generally. In Appendix C we consider an alternative calibration that deviates from the benchmark in two key ways.

First, we consider significantly lower deposit spreads, treating deposits as more akin to shadow banking activity. van Binsbergen, Diamond, and Grotteria (2022) find convenience yields on Treasuries in the 35-45bps range, which we use as a target for the convenience yields for financial sector liabilities. Second, we assume that banks' only advantage comes from their ability to issue liquid liabilities. This is consistent with the evidence in Egan, Lewellen, and Sunderam (2021) who find that 70% of the variation in bank value comes from deposit creation, not asset issuance, suggesting that bank value is driven by their liabilities and not by investment advantages. This would be consistent with shadow banking activity in which institutions hold tradable securities to issue short-term debt—without enhancing the payoffs of the financial securities they hold.

Even though banks are much less important for liquidity provision in this alternative calibration, and are not at all important for the capital allocation, the overall results of our paper continue to hold in that very different calibration with some quantitative differences. First, the potential welfare gains are lower, with gains near 0.6% consumption-equivalent for $m = 18.27\%$. Second, the consequences for financial stability are much greater. With $m = 18.27\%$, crises are more than 5 times more frequent and bank valuations decline by nearly 20%. In this case, because banks have smaller advantages to start, the addition of digital currency is more destabilizing for the financial system and provides fewer potential benefits.

Traditional Banking and Stablecoin Issuance In our model financial intermediaries produce liquidity by holding real risky assets. The "cost" of intermediation arises due to financial frictions (the need to avoid bankruptcy) but there are otherwise no direct costs of intermediation (banks pay a cost $\mathscr{T}_b$, but this is not a resource cost that decreases output as does ι, the cost of issuing digital currency). In contrast, digital currencies are issued by holding risk-free assets at an assumed cost. Even though the costs κ can be quite small, they are still non-negligible.

How should we interpret the costs κ? In the case of a CBDC, this is likely capturing risks to government funding that occur when the government substantially shortens the maturity of its borrowing, and could potentially reflect inflationary risks. In the case of privately issued digital currency, the cost could reflect the real costs of maintaining the infrastructure of a stable coin (managing off-chain collateral) and potential protocol risks. It's possible that the costs of issuing digital currency are actually much smaller than we've assumed. In that case, our positive results maintain, but the welfare gains of issuing digital currency will natural increase.

In our model, banks do not have the ability to issue stablecoins as part of their portfolio of offerings. There are a few ways to potentially justify this assumption. First, banks may be prohibited due to regulation. Second, the profitability of issuing digital currency is at best on the order of basis points and yet is a very balance-sheet intensive activity. Regulatory constraints such as the Supplementary Leverage Ratio may make issuance of digital currency backed 100% by risk-free assets prohibitively expensive (indeed, there are many near-arbitrages of similar profitability, such as CIP violations, that banks avoid due to balance sheet considerations). Finally, in some cases the optimal level of m occurs when digital currency issuance loses money on average. In this case, banks would optimally choose not to enter the market.

5 Conclusion

Digital currencies have the potential to greatly reshape the financial sector. We provide a macroeconomic model with a financial sector in which digital currencies coexist with bank deposits and households hold both forms of liquidity. Our main theoretical result is that when banks face financial frictions (costly equity issuance), digital currency harms financial stability, increasing the likelihood of crises and financial distress. Digital currencies depress deposit spreads, which hinders banks' abilities to recapitalize following losses. Digital currency, whether privately or publicly issued, is likely to be detrimental for financial stability and bank valuations can be significantly harmed. Despite the costs to financial stability, we find that digital currency can be welfare improving for households. In our benchmark calibration, the welfare-maximizing level of digital currency increases household welfare by 2.8% of consumption-equivalent even as the probability of crises doubles from 3 to 6%. Our results suggest that financial frictions may limit the potential benefits of digital currencies, and the optimal level of digital currency may be below what would be issued in a competitive environment.

References

ACHARYA, V. V., I. GUJRAL, N. KULKARNI, AND H. S. SHIN (2011): "Dividends and bank capital in the financial crisis of 2007-2009," Tech. rep., National Bureau of Economic Research.

ADRIAN, T. AND N. BOYARCHENKO (2012): "Intermediary leverage cycles and financial stability," Tech. Rep. 567, Staff Report, Federal Reserve Bank of New York.

ADRIAN, T. AND H. SHIN (2010): "Liquidity and Leverage," *Journal of Financial Intermediation*, 19, 418–437.

ADRIAN, T. AND H. S. SHIN (2014): "Procyclical leverage and value-at-risk," *Review of Financial Studies*, 27, 373–403.

AHNERT, T., K. ASSENMACHER, P. HOFFMANN, A. LEONELLO, C. MONET, AND D. PORCELLACCHIA (2022a): "The economics of central bank digital currency," .

AHNERT, T., P. HOFFMANN, A. LEONELLO, AND D. PORCELLACCHIA (2020): "Central Bank Digital Currency and Financial Fragility," Tech. rep., mimeo.

AHNERT, T., P. HOFFMANN, AND C. MONET (2022b): "The digital economy, privacy, and CBDC," .

BEGENAU, J. AND T. LANDVOIGT (2021): "Financial Regulation in a Quantitative Model of the Modern Banking System," *The Review of Economic Studies*, rdab088.

BERNANKE, B. S., M. GERTLER, AND S. GILCHRIST (1999): "The financial accelerator in a quantitative business cycle framework," in *Handbook of Macroeconomics*, ed. by J. B. Taylor and M. Woodford, Elsevier, vol. 1 of *Handbook of Macroeconomics*, chap. 21, pp. 1341–1393.

BRUNNERMEIER, M. K. AND Y. SANNIKOV (2014): "A Macroeconomic Model with a Financial Sector," *American Economic Review*, 104, 379–421.

——— (2015): "International Credit Flows and Pecuniary Externalities," *American Economic Journal: Macroeconomics*, 7, 297–338.

——— (2016a): "The I theory of money," Tech. rep., National Bureau of Economic Research.

——— (2016b): "Macro, money, and finance: A continuous-time approach," in *Handbook of Macroeconomics*, Elsevier, vol. 2, 1497–1545.

BURLON, L., C. MONTES-GALDON, M. MUÑOZ, AND F. SMETS (2022): "The optimal quantity of CBDC in a bank-based economy," .

CHEN, W. AND G. PHELAN (2022a): "Liquidity Provision and Financial Stability," *Journal of Money, Credit and Banking, forthcoming*.

——— (2022b): "Should Monetary Policy Target Financial Stability?" *Review of Economic Dynamics, forthcoming*.

D'AVERNAS, A., T. BOURANY, AND Q. VANDEWEYER (2022): "Can Stablecoins be Stable?" Tech. rep., Working Paper.

DI TELLA, S. (2017): "Uncertainty Shocks and Balance Sheet Recessions," *Journal of Political Economy*, 125, 2038 – 2081.

——— (2020): "Risk premia and the real effects of money," *American Economic Review*, 110, 1995–2040.

DIAMOND, D. W. AND P. H. DYBVIG (1983): "Bank Runs, Deposit Insurance, and Liquidity," *The Journal of Political Economy*, 91, pp. 401–419.

DRECHSLER, I., A. SAVOV, AND P. SCHNABL (2017): "The deposits channel of monetary policy," *The Quarterly Journal of Economics*, 132, 1819–1876.

——— (2018): "A model of monetary policy and risk premia," *The Journal of Finance*, 73, 317–373.

EGAN, M., S. LEWELLEN, AND A. SUNDERAM (2021): "The Cross-Section of Bank Value," *The Review of Financial Studies*, hhab089.

EUROPEAN CENTRAL BANK (2020): "Report on a digital euro," Tech. rep., European Central Bank.

FEDERAL RESERVE (2022): "Money and Payments: The US Dollar in the Age of Digital Transformation," *January, https://www. federalreserve. gov/publications/moneyand-payments-discussion-paper. html*.

FERNÁNDEZ-VILLAVERDE, J., D. SANCHES, L. SCHILLING, AND H. UHLIG (2021): "Central bank digital currency: Central banking for all?" *Review of Economic Dynamics*, 41, 225–242.

GALI, J. (2008): *Monetary policy, inflation and the business cycle: An introduction to the New Keynesian framework*, Princeton: Princeton University Press.

GORTON, G. AND G. PENNACCHI (1990): "Financial Intermediaries and Liquidity Creation," *Journal of Finance*, 45, 49–71.

GORTON, G. B., C. P. ROSS, AND S. Y. ROSS (2022): "Making Money," NBER Working paper No. 29710.

HE, Z., B. KELLY, AND A. MANELA (2017): "Intermediary asset pricing: New evidence from many asset classes," *Journal of Financial Economics*, 126, 1–35.

HE, Z. AND A. KRISHNAMURTHY (2012): “A Model of Capital and Crises,” *The Review of Economic Studies*, 79, 735–777.

——— (2013): “Intermediary Asset Pricing,” *American Economic Review*, 103, 732–70.

——— (2019): “A macroeconomic framework for quantifying systemic risk,” *American Economic Journal: Macroeconomics*, 11, 1–37.

JORDÀ, O., M. SCHULARICK, AND A. M. TAYLOR (2013): “When Credit Bites Back,” *Journal of Money, Credit and Banking*, 45, 3–28.

KEISTER, T. AND C. MONNET (2022): “Central bank digital currency: Stability and information,” *Journal of Economic Dynamics and Control*, 104501.

KEISTER, T. AND D. R. SANCHES (2021): “Should central banks issue digital currency?” *Available at SSRN 3966817*.

KIYOTAKI, N. AND J. MOORE (1997): “Credit Cycles,” *Journal of Political Economy*, 105, pp. 211–48.

LAGOS, R. AND R. WRIGHT (2005): “A Unified Framework for Monetary Theory and Policy Analysis,” *Journal of Political Economy*, 113, 463–484.

MAGGIORI, M. (2017): “Financial Intermediation, International Risk Sharing, and Reserve Currencies,” *American Economic Review*, 107, 3038–71.

MOREIRA, A. AND A. SAVOV (2017): “The Macroeconomics of Shadow Banking,” *Journal of Finance*.

PHELAN, G. (2016): “Financial intermediation, leverage, and macroeconomic instability,” *American Economic Journal: Macroeconomics*, 8, 199–224.

SKEIE, D. (2021): “Digital Currency Runs,” Tech. rep., WBS Finance Group Research Paper.

VAN BINSBERGEN, J. H., W. F. DIAMOND, AND M. GROTTERIA (2022): “Risk-free interest rates,” *Journal of Financial Economics*, 143, 1–29.

Van der Ghote, A. (2019): "Liquidity Management, Leverage, and Monetary Policy," Tech. rep., European Central Bank.

——— (2021): "Interactions and Coordination between Monetary and Macroprudential Policies," *American Economic Journal: Macroeconomics*, 13, 1–34.

Whited, T. M., Y. Wu, and K. Xiao (2022): "Central Bank Digital Currency and Banks," *Available at SSRN 4112644*.

Williamson, S. D. (2021): "Central bank digital currency and flight to safety," *Journal of Economic Dynamics and Control*, 104146.

——— (2022): "Central bank digital currency: Welfare and policy implications," *Journal of Political Economy, forthcoming*.

Appendices

A Proofs and Additional Propositions

Household Optimality

Proof of Proposition 1, households' optimality decisions. We characterize the household problem as follows. Conjecture that households have a twice-differentiable value function

$$V_t = \frac{\log(w_{h,t})}{r} + h_t,$$

where h_t is independent of the level of household wealth. By Ito's lemma, households' HJB is

$$\begin{aligned}\log(w_{h,t}) + rh_t = &\max_{x_t \geq 0,\, c_{h,t}} \log(c_{h,t}) + \beta \log(\ell_{h,t}) \\ &+ \frac{1}{rw_{h,t}} \mu_{wh,t} w_{h,t} - \frac{1}{2r(w_{h,t})^2} (\sigma_{wh,t} w_{h,t})^2 \\ = &\max_{x_t \geq 0,\, c_{h,t}} \log(c_{h,t}) + \beta \log(\ell_{h,t}) + \frac{1}{r} \mu_{wh,t} - \frac{1}{2r} \sigma^2_{wh,t}.\end{aligned}$$

Further, because $\sigma_{wh,t} = x_{h,t}(\sigma + \sigma_{Q,t}) + x_{e,t}\sigma_{e,t}$,

$$\frac{\partial \sigma_{wh,t}}{\partial x_{h,t}} = (\sigma + \sigma_{Q,t}), \quad \frac{\partial \sigma_{wh,t}}{\partial x_{e,t}} = \sigma_{e,t}.$$

Finally, households' law of motion for their net worth can also be written as

$$\begin{aligned}\frac{dw_{h,t}}{w_{h,t}} = &dr_{f,t} + x_{h,t}(dr_{h,t} - dr_{f,t}) + x_{e,t}(dr_{e,t} - dr_{f,t}) \\ &+ x_{d,t}(dr_{d,t} - dr_{f,t}) + x_{dc,t}(dr_{dc,t} - dr_{f,t}) + \left(\mathscr{T}_{h,t} - \frac{c_{h,t}}{w_{h,t}} \right) dt.\end{aligned}$$

Hence, wealth grows at the risk-free rate $r_{f,t}$ plus the spread or risk-premium determined by allocations in the other assets.

Note that total deposits and digital currency are given by $x_{d,t}w_{h,t}$ and $x_{dc,t}w_{h,t}$. Given these observations and (6), the first-order conditions are

$$
\begin{aligned}
(x_{h,t}): \quad 0 &= \frac{1}{r}(\mathbb{E}[dr_{h,t}] - r_{f,t}) - \frac{1}{r}\sigma_{wh,t}\frac{\partial \sigma_{wh,t}}{\partial x_{h,t}} \\
&= (\mathbb{E}[dr_{h,t}] - r_{f,t} - \sigma_{wh,t}(\sigma + \sigma_{Q,t})) \\
\Rightarrow \mathbb{E}[dr_{h,t}] - r_{f,t} &= \sigma_{wh,t}(\sigma + \sigma_{Q,t}) \\
(x_{e,t}): \quad 0 &= \frac{1}{r}(\mathbb{E}[dr_{e,t}] - r_{f,t}) - \frac{1}{r}\sigma_{wh,t}\frac{\partial \sigma_{wh,t}}{\partial x_{e,t}} \\
&= \mathbb{E}[dr_{e,t}] - r_{f,t} - \sigma_{wh,t}\sigma_{e,t} \\
\Rightarrow \mathbb{E}[dr_{e,t}] - r_{f,t} &= \sigma_{wh,t}\sigma_{e,t}, \\
(x_{d,t}): \quad 0 &= v'(\ell_{h,t})\frac{\partial \ell}{\partial x_{d,t}}w_{h,t} + \frac{1}{r}(r_{d,t} - r_{f,t}) \\
\Rightarrow r_{d,t} &= r_{f,t} - rv'(\ell_{h,t})\frac{\partial \ell}{\partial x_{d,t}}w_{h,t}
\end{aligned}
$$

$$
\begin{aligned}
(x_{dc,t}): \quad 0 &= v'(\ell_{h,t})\frac{\partial \ell}{\partial x_{dc,t}}w_{h,t} + \frac{1}{r}(r_{dc,t} - r_{f,t}) \\
\Rightarrow r_{dc,t} &= r_{f,t} - rv'(\ell_{h,t})\frac{\partial \ell}{\partial x_{dc,t}}w_{h,t} \\
(c_{h,t}): \quad 0 &= \frac{1}{c_{h,t}} - \frac{1}{rw_{h,t}}, \\
\Rightarrow c_{h,t} &= rw_{h,t}.
\end{aligned}
$$

Since consumption is proportional to wealth,

$$
\sigma_{ch,t} = x_{h,t}(\sigma + \sigma_{Q,t}) + x_{e,t}\sigma_{e,t}. \tag{23}
$$

Hence the first-order conditions for $x_{h,t}$ and $x_{e,t}$ equate expected excess returns with the covariance of household consumption and returns.

□

Risk-free Rate

Lemma 2. *Generally, the equilibrium risk-free rate is*

$$dr_{f,t} = \left(r + \mu_{ch,t} - \sigma_{ch,t}^2 - \mathscr{T}_{wh,t}\right) dt,$$

where $\mathscr{T}_{wh,t}$ are household transfers that are proportional to household wealth. Thus, in the dynamic model, the risk-free rate is given by

$$r_{f,t} = r + g_{y,t} - \sigma^2,$$

since government taxes are lump-sum and banks pay dividends only at the boundary. In the first-best equilibrium, the risk-free rate is

$$r_{f,FB} = r + g_b - \sigma^2 + DS_{FB},$$

reflecting net bank dividends.

Proof. To prove this claim, we employ the stochastic maximum principle (see Brunnermeier and Sannikov (2016b)). Let $\mathscr{T}_{wh,t}$ denote transfers proportional to household wealth and let $\mathscr{T}_{LS,t}$ denote lump-sum transfers that are not proportional to wealth. Let $\hat{x}_t = (x_{y,t}, x_{d,t}, x_{dc,t}, x_{e,t})$ be household's portfolio weight on capital, deposits, digital currency, and bank equity respectively (i.e., not risk-free bonds). The Hamiltonian is

$$\mathscr{H}_t = e^{-rt} \log(c_{h,t}) + \xi_t \mu_{wh,t} w_{h,t} - \varsigma_t \xi_t \sigma_{wh,t} w_{h,t}$$

where the drifts and volatilities are

$$\begin{aligned}
\mu_{wh,t} w_{h,t} &= w_{h,t} \left(dr_{f,t} + \mathscr{T}_{wh,t} + \hat{x}_t \cdot (\mathbb{E}[dr_t - dr_{f,t}])\right) - c_{h,t} + \mathscr{T}_{LS,t} \\
\sigma_{wh,t} w_{h,t} &= w_{h,t} \left(x_{h,t}(\sigma + \sigma_{Q,t}) + x_{e,t} \sigma_{e,t}\right).
\end{aligned}$$

The FOCs with respect to $c_{h,t}$ is

$$0 = e^{-rt}\frac{1}{c_{h,t}} - \xi_t,$$

which implies that $\xi_t = e^{-rt}/c_{h,t}$, hence by Ito's lemma

$$\frac{d\xi_t}{\xi_t} = (-r - \mu_{ch,t} + \sigma_{ch,t}^2)\,dt - \sigma_{ch,t}\,dW_t.$$

By the stochastic maximum principle,

$$\begin{aligned}
\mu_{\xi,t}\xi_t &= -\frac{\partial}{\partial w_{h,t}}\mathscr{H}_t \\
&= -\xi_t\big(\big(dr_{f,t} + \mathscr{T}_{wh,t} + \hat{x}_t \cdot (\mathbb{E}[dr_t - dr_{f,t}])\big) - \varsigma_t \sigma_{wh,t}\big) \\
\varsigma_{\xi,t} &= -\sigma_{ch,t}
\end{aligned}$$

Using the above equation, together with the the FOCs from Proposition 1, and the implications of the stochastic maximum principle for $\mu_{\xi,t}$ and $\varsigma_{\xi,t}$, we obtain

$$\mu_{\xi,t} = -(\mathbb{E}[dr_{f,t}] + \mathscr{T}_{wh,t}).$$

Substitute in for $\mu_{\xi,t}$ to acquire

$$\begin{aligned}
-(r + \mu_{ch,t} - \sigma_{ch,t}^2) &= -(\mathbb{E}[dr_{f,t}] + \mathscr{T}_{wh,t}) \\
\mathbb{E}[dr_{f,t}] &= r + \mu_{ch,t} - \sigma_{ch,t}^2 - \mathscr{T}_{wh,t},
\end{aligned}$$

and hence $dr_{f,t} = (r + \mu_{ch,t} - \sigma_{ch,t}^2 - \mathscr{T}_{wh,t})dt$.

In the dynamic equilibrium we have $\mathscr{T}_{wh,t} = 0$, $\mu_{ch,t} = g_{y,t}$, and $\sigma_{ch,t} = \sigma$, which yields the equation for the risk-free rate in the dynamic equilibrium.

In the first-best banks hold the entire tree and fund themselves with deposits, and so the tree is priced relative to the deposit rate. In the first-best (which is stationary, the banks' portfolio condition can be written

$$\mathbb{E}[dr_b] - r_{d,FB} = (\sigma)(\sigma),$$

or

$$\frac{1}{Q_{FB}} + g_b - r_{f,FB} = \sigma^2 - DS_{FB}.$$

Rearranging the banks' condition yields

$$Q_{FB} = \frac{1}{r_{f,FB} - g_b + \sigma^2 - DS_{FB}}.$$

Recall that the first-best price is $Q_{FB} = \frac{1}{r}$, which implies

$$r_{d,FB} = r + g_b - \sigma^2, \quad r_{f,FB} = r + g_b - \sigma^2 + DS_{FB}.$$

$\square$

Bank Optimality

Proof of Proposition 2, banks' optimality decisions. Homogeneity and price-taking imply that banks' value function takes the form $U_t = \theta_t n_{b,t}$, where θ_t is the marginal value of banks' equity. Further, households' discount factor is $\xi_t = e^{-rt} c_{h,t}^{-1}$ because households have log utility. Therefore, the HJB can be written as

$$r\theta_t n_{b,t} c_{h,t}^{-1} = \max_{y_{b,t} \geq 0, d\zeta_t, \iota_t} d\zeta_t c_{h,t}^{-1} + \mathbb{E}[d(\theta_t n_{b,t} c_{h,t}^{-1})], \tag{24}$$

subject to the budget and net-worth constraints. Conjecture that θ_t and $c_{h,t}$ both follow diffusions. By Ito's product rule,

$$\frac{d(\theta_t n_{b,t})}{\theta_t n_{b,t}} = (\mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t}\sigma_{nb,t})\, dt + (\sigma_{\theta,t} + \sigma_{nb,t})\, dW_t.$$

By Ito's product and quotient rules,

$$(\theta_t n_{b,t} c_{h,t}^{-1})\mathbb{E}\left[\frac{d(\theta_t n_{b,t} c_{h,t}^{-1})}{\theta_t n_{b,t} c_{h,t}^{-1}}\right]$$
$$= \mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t}\sigma_{nb,t} + \sigma_{ch,t}^2 - \mu_{ch,t} - \sigma_{ch,t}(\sigma_{\theta,t} + \sigma_{nb,t}).$$

After dropping the differential dt, equation (24) becomes

$$r\theta_t n_{b,t} c_{h,t}^{-1} = \max_{y_{b,t}\geq 0, \iota_t, d\zeta_{b,t}\geq 0} c_{h,t}^{-1} d\zeta_{b,t} + \theta_t n_{b,t} c_{h,t}^{-1}(\mu_{\theta,t} + \mu_{nb,t} - \mu_{ch,t} + \sigma_{ch,t}^2$$
$$+ \sigma_{\theta,t}\sigma_{nb,t} - \sigma_{ch,t}\sigma_{nb,t} - \sigma_{ch,t}\sigma_{\theta,t}).$$

Dividing through by $\theta_t n_{b,t} c_{h,t}^{-1}$, substituting wealth terms we have

$$r = \max \frac{d\zeta_{b,t}}{\theta_t n_{b,t}} + \mu_{\theta,t} + r_{d,t} + x_{b,t}(\mathbb{E}[dr_{b,t}] - r_{d,t}) - \frac{d\zeta_{b,t}}{n_{b,t}} - \mathscr{T}_{b,t}$$
$$- \mu_{ch,t} + \sigma_{ch,t}^2 + x_{b,t}\sigma_{\theta,t}(\sigma + \sigma_{Q,t}) - x_{b,t}\sigma_{ch,t}(\sigma + \sigma_{Q,t}) - \sigma_{ch,t}\sigma_{\theta,t}$$

Unlike the household problem, we may plug this constraint directly into the problem because banks do not have money-in-the-utility demand or costs from raising bank deposits which would introduce spreads into the bank problem.

The risk-free interest rate satisfies $r_{f,t} = r + \mu_{ch,t} - \sigma_{ch,t}^2$. Adding $\mu_{ch,t} - \sigma_{ch,t}^2$ to both sides of the HJB and re-arranging:

$$r_{f,t} = \max \frac{d\zeta_{b,t}}{n_{b,t}}\left(\frac{1}{\theta_t} - 1\right) + \mu_{\theta,t} - \sigma_{ch,t}\sigma_{\theta,t} + r_{f,t} + r_{d,t} - r_{f,t}$$
$$+ x_{b,t}(\mathbb{E}[dr_{b,t}] - r_{d,t} + (\sigma_{\theta,t} - \sigma_{ch,t})(\sigma + \sigma_{Q,t})) - \mathscr{T}_{b,t}$$
$$r_{f,t} - r_{d,t} = \max \frac{d\zeta_{b,t}}{n_{b,t}}\left(\frac{1}{\theta_t} - 1\right) + \mu_{\theta,t} - \sigma_{ch,t}\sigma_{\theta,t} - \mathscr{T}_{b,t}$$
$$+ x_{b,t}(\mathbb{E}[dr_{b,t}] - r_{d,t} + (\sigma_{\theta,t} - \sigma_{ch,t})(\sigma + \sigma_{Q,t})).$$

The first-order conditions imply

$$d\zeta_{b,t} > 0 \text{ if } \theta_t \leq 1; \quad d\zeta_{b,t} = 0 \text{ if } \theta_t \in (1, 1+\gamma); \quad d\zeta_{b,t} < 0 \text{ if } \theta_t \geq 1+\gamma;$$
$$\mathbb{E}[dr_{b,t}] - r_{d,t} = (\sigma_{ch,t} - \sigma_{\theta,t})(\sigma + \sigma_{Q,t}).$$

The FOC on asset holdings can be re-written as

$$\mathbb{E}[dr_{b,t}] - r_{f,t} = (\sigma_{ch,t} - \sigma_{\theta,t})(\sigma + \sigma_{Q,t}) - (r_{f,t} - r_{d,t}).$$

Plugging these first-order conditions into the HJB and re-arranging yields

$$\mu_{\theta,t} = r_{f,t} - r_{d,t} + \sigma_{ch,t}\sigma_{\theta,t} + \mathscr{T}_{b,t}.$$

Finally, it remains to price traded bank equity shares. To start, the return to bank equity includes dividends paid $d\zeta_t$ and capital gains. The aggregate value of bank dividends, discounted by the household stochastic discount factor, is $U_t = \theta_t N_{b,t}$. Banks pay dividends only when $\theta_t = 1$. Since the dividend payout decreases bank wealth by exactly the amount paid, this means that the value $U_t = \theta_t N_{b,t}$ does not change when dividends are paid (there is an identical offsetting term in the capital gain). Because consumption is proportional to wealth, banks' HJB implies that

$$r + \mu_{ch,t} - \sigma_{ch,t}^2 = \mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t}\sigma_{nb,t} - \sigma_{ch,t}\sigma_{nb,t} - \sigma_{ch,t}\sigma_{\theta,t}.$$

It follows that

$$r_{f,t} = \mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t}\sigma_{nb,t} - \sigma_{ch,t}(\sigma_{nb,t} + \sigma_{\theta,t}),$$

and so

$$\mathbb{E}[\mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t}\sigma_{nb,t} - r_{f,t}] = \sigma_{ch,t}(\sigma_{\theta,t} + \sigma_{nb,t}). \tag{25}$$

Then the market price of traded bank shares equals $\theta_t N_{b,t}$ so that $\mathbb{E}[\mu_{\theta,t}+\mu_{nb,t}+\sigma_{\theta,t}\sigma_{nb,t}-r_{f,t}] = \sigma_{ch,t}(\sigma_{\theta,t}+\sigma_{nb,t})$, where $\mathbb{E}[dr_{e,t}] = \mu_{\theta,t}+\mu_{nb,t}+\sigma_{\theta,t}\sigma_{nb,t}$ and $\sigma_{e,t} = \sigma_{\theta,t}+\sigma_{nb,t}$. The expected return on equity includes expected changes in the book equity $\mu_{nb,t}$ and expected changes in the marginal valuation $\mu_{\theta,t}+\sigma_{\theta,t}\sigma_{nb,t}$, where the last term is the Ito interaction term. We need not include the dividend yield $d\zeta_t$ directly for the reasons just given. In addition, the volatility of returns is given by $\sigma_{e,t} = \sigma_{\theta,t}+\sigma_{nb,t}$. From households' optimization over bank equity, $\mathbb{E}[dr_{e,t}] - r_{f,t} = \sigma_{ch,t}\sigma_{e,t}$. □

Characterizing Equilibrium

Proof of Lemma 1, law of motion for η_t. After substituting banks' asset pricing condition and using $x_{b,t} = \psi_t/\eta_t$, the process for banks' aggregate equity $N_{b,t}$ is

$$\begin{aligned}\frac{dN_{b,t}}{N_{b,t}} &= \left(r_{d,t}+\frac{\psi_t}{\eta_t}(\sigma_{ch,t}-\sigma_{\theta,t})(\sigma+\sigma_{Q,t})-\mathscr{T}_{b,t}\right)dt-\frac{d\zeta_{b,t}}{N_{b,t}}+\frac{\psi_t}{\eta_t}(\sigma+\sigma_{Q,t})\,dW_t\\ &= \left(r_{d,t}+\frac{\psi_t}{\eta_t}(\sigma_{ch,t}-\sigma_{\theta,t})(\sigma+\sigma_{Q,t})-\mathscr{T}_{b,t}\right)dt-\frac{d\zeta_{b,t}}{N_{b,t}}+\frac{\psi_t}{\eta_t}(\sigma+\sigma_{Q,t})\,dW_t.\end{aligned}$$

The law of motion for the value of the tree is

$$\frac{d(Q_tY_t)}{Q_tY_t} = (\mu_{Q,t}+g_{y,t}+\sigma\sigma_{Q,t})\,dt+(\sigma+\sigma_{Q,t})\,dW_t.$$

Because banks' asset pricing condition always holds in equilibrium, we may write

$$\begin{aligned}\mu_{Q,t} &= r_{f,t}-DS_t+(\sigma_{ch,t}-\sigma_{\theta,t})(\sigma+\sigma_{Q,t})-\frac{1}{Q_t}-g_{y,t}-\sigma\sigma_{Q,t},\\ &= r-DS_t-\sigma_{\theta,t}(\sigma+\sigma_{Q,t})-\frac{1}{Q_t}.\end{aligned}$$

Substituting into the law of motion for the value of capital

$$\frac{d(Q_tY_t)}{Q_tY_t} = \left(r-DS_t-\sigma_{\theta,t}(\sigma+\sigma_{Q,t})-\frac{1}{Q_t}+g_{y,t}+\sigma\sigma_{Q,t}\right)dt+(\sigma+\sigma_{Q,t})\,dW_t.$$

By Ito's quotient rule,

$$\frac{d(1/(Q_tY_t))}{1/(Q_tY_t)} = \left((\sigma+\sigma_{Q,t})^2 - \left(r - DS_t - \sigma_{\theta,t}(\sigma+\sigma_{Q,t}) - \frac{1}{Q_t} + g_{y,t} + \sigma\sigma_{Q,t}\right)\right) dt - (\sigma+\sigma_{Q,t})\, dW_t.$$

Using Ito's product rule,

$$\begin{aligned}\frac{d\eta_t}{\eta_t} &= \left(r_{d,t} + \frac{\psi_t}{\eta_t}(\sigma_{ch,t}-\sigma_{\theta,t})(\sigma+\sigma_{Q,t}) - \mathscr{T}_{b,t}\right) dt - \frac{d\zeta_{b,t}}{N_{b,t}} \\ &\quad + \left((\sigma+\sigma_{Q,t})^2 - \left(r - DS_t - \sigma_{\theta,t}(\sigma+\sigma_{Q,t}) - \frac{1}{Q_t} + g + \sigma\sigma_{Q,t}\right)\right) dt - (\sigma+\sigma_{Q,t})\, dt \\ &\quad - \frac{\psi_t}{\eta_t}(\sigma+\sigma_{Q,t})^2\, dt + \left(\frac{\psi_t}{\eta_t} - 1\right)(\sigma+\sigma_{Q,t})\, dW_t.\end{aligned}$$

The drift simplifies to

$$\mu_{\eta,t} = \left(\frac{\psi_t}{\eta_t} - 1\right)((\sigma_{ch,t}-\sigma_{\theta,t}) - (\sigma+\sigma_{Q,t}))(\sigma+\sigma_{Q,t}) - \mathscr{T}_{b,t} + \frac{1}{Q_t} - \frac{d\zeta_{b,t}}{N_{b,t}}.$$

Define $d\Xi_t \equiv d\zeta_{b,t}/N_{b,t}$ as a control creating reflecting barriers. The drift and volatility of $d\eta_t/\eta_t$ now match (14).

□

Proof of Proposition 3. The proposition has the same proof as the analogous statements in Phelan (2016); Chen and Phelan (2022b). We derive the smooth pasting condition here. Suppose there exists a market for claims on the aggregate capital stock. Arbitrage pricing for the risky asset requires that

$$Q_tY_t = \mathbb{E}_t\left[\int_t^\infty \frac{\xi_\tau}{\xi_t}\mathrm{Div}_\tau\, d\tau\right] = C_{h,t}\mathbb{E}_t\left[\int_t^\infty e^{-r(s-t)}\frac{\mathrm{Div}_s}{C_{h,\tau}}\, d\tau\right]$$

where Div_t are dividends from capital. Define $G_t \equiv Q_tY_t/C_{h,t}$. Then the previous equation simplifies to

$$G_t = \mathbb{E}_t\left[\int_t^\infty e^{-r(\tau-t)}\frac{\mathrm{Div}_s}{C_{h,\tau}}\, d\tau\right],$$

which implies the equation

$$rG_t = \frac{\mathrm{Div}_t}{C_{h,t}} + \mathbb{E}[dG_t].$$

Drop time subscripts to reduce clutter and plug in for household consumption.

$$G \equiv \frac{QY}{C_h} = \frac{QY}{rq(1+(\theta-1)\eta)K} = \frac{1}{r(1+(\theta-1)\eta)}.$$

If there is a reflecting barrier at $\eta = \eta^R$, then smooth pasting requires

$$0 = \frac{\partial}{\partial \eta} G(\eta^R) = -G \frac{\theta'(\eta^R)\eta^R + \theta(\eta^R) - 1}{1 + (\theta(\eta^R) - 1)\eta^R}.$$

Note that this condition is equivalent to requiring $Q'(\eta^R) = 0$, which is the standard smooth pasting condition for an asset price. The derivative of G with respect to η captures the marginal benefit of increasing η at the barrier, and it should equal the marginal cost of the barrier. However, because banks pay the cost of reflection, someone holding a claim to the aggregate capital stock faces zero marginal cost. Therefore, the smooth-pasting condition for Q_t at a barrier η^R is

$$\theta'(\eta^R)\eta^R + \theta(\eta^R) = 1. \tag{26}$$

□

First Best

Proof of Proposition 5. Define $\hat{\ell} = \left((Q_{FB})^{\frac{\varepsilon-1}{\varepsilon}} + \delta \left(mq^D \right)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}$. Then $\ell_{FB} = \frac{r}{1-\iota}\hat{\ell}$. Note that $\left(\frac{\ell}{x_{dc}}\right) = \left(\frac{\hat{\ell}}{mq^D}\right)$. Maximizing welfare is equivalent to choosing m to maximize

$$\log(1-\iota) + \beta \log(\hat{\ell}) - \beta \log(1-\iota).$$

For an interior solution, the optimal m^* solves

$$\begin{aligned}
\frac{\kappa q^D}{1-\iota} &= \frac{\beta}{\hat{\ell}} \frac{\partial \hat{\ell}}{\partial (mq^D)} q^D + \frac{\beta \kappa q^D}{1-\iota}, \\
\frac{\kappa q^D}{1-\iota} &= \frac{r\beta}{\ell(1-\iota)} \frac{\partial \ell}{\partial x_{dc}} q^D + \frac{\beta \kappa q^D}{1-\iota}, \\
\kappa &= \frac{r\beta}{\ell} \frac{\partial \ell}{\partial x_{dc}} + \beta \kappa, \\
\kappa &= DCS + \beta \kappa.
\end{aligned}$$

For further intuition, note that

$$\frac{\partial x_{dc}}{\partial m} = rq^D \frac{(1-\iota)(1) - (m)(-\kappa q^D)}{(1-\iota)^2} = \frac{rq^D}{(1-\iota)^2} > 0,$$

and

$$\frac{\partial x_d}{\partial m} = \frac{rQ_{FB} \kappa q^D}{(1-\iota)^2} = \frac{\kappa q^D}{(1-\iota)^2} > 0.$$

Digital currency increases digital currency directly, but also increase the fraction of wealth held in deposits and digital currency, thus providing a second-round effect increasing liquidity services.

□

B Additional Results

This section provides results underlying the analysis in the main text, or additional robustness results beyond the analysis in the main text.

B.1 Tables

Table 6 provides the equilibrium consequences of introducing digital currency but setting the currency preference to $\delta = 0.5$ and $\kappa = 50bps$. The other parameters match the baseline calibration with $\varepsilon = 5.3$. The qualitative results match the baseline results in the paper, though the quantitative significance decreases (crisis probability increases by less, welfare gains are smaller, decrease in bank valuations are smaller, etc.)

Table 6: Model outcomes with baseline calibration, $\varepsilon = 5.3$ with $\delta = 0.5$. Averages of variables are computed by integrating the moment of interest with respect to the stationary density.

Moment	**Baseline** $m = 0$	$m = 18.27\%$	$m = 26.1\%$
Average Deposit Rate	108.6 bps	90.6 bps	85.4 bps
Average Bank Leverage	10.11	10.71	10.96
Prob. of Crisis ($\psi < 50\%$)	2.99%	4.36%	4.98%
Prob. of Distress ($\psi < 100\%$)	65.77%	68.43%	69.39%
Average Sharpe Ratio	47.26%	47.25%	47.36%
Output Loss in Crisis	-8.04%	-8.48%	-8.61%
Average Bank Holdings ψ	87.03%	85.18%	84.51%
Average Bank Equity η	11.52%	10.83%	10.61%
Asset Price Volatility σ_Q	3.19%	2.96%	2.89%
System Drift $\eta\mu_\eta$	0.20%	0.19%	0.18%
System Volatility $\eta\sigma_\eta$	4.20%	3.98%	3.91%
Welfare Gains (CE)	–	1.76%	1.86%
Change in Bank Valuations (pct)	–	-3.56%	-4.76%

Table 7 provides the equilibrium consequences of introducing digital currency with a high elasticity between digital currency and deposits, setting $\varepsilon = 53$. The other parameters match the baseline calibration. The results are comparable to the benchmark results with $\varepsilon = 5.3$ except that in this case with high substitutibility welfare decreases at these levels of m.

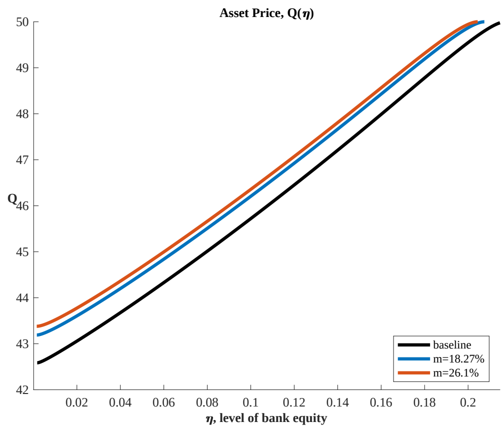
Table 7: Model outcomes with baseline calibration and high elasticity, $\varepsilon = 53$. Averages of variables are computed by integrating the moment of interest with respect to the stationary density.

Moment	**Baseline** $m = 0$	$m = 18.27\%$	$m = 26.1\%$
Average Deposit Rate	108.6 bps	80.5 bps	71.5 bps
Average Bank Leverage	10.11	11.22	11.64
Prob. of Crisis ($\psi < 50\%$)	2.99%	5.64%	6.79%
Prob. of Distress ($\psi < 100\%$)	65.77%	70.06%	71.58%
Average Sharpe Ratio	47.26%	47.43%	47.49%
Output Loss in Crisis	-8.01%	-8.72%	-8.96%
Average Bank Holdings ψ	87.03%	83.85%	82.64%
Average Bank Equity η	11.52%	10.39%	9.99%
Asset Price Volatility σ_Q	3.19%	2.81%	2.68%
System Drift $\eta\mu_\eta$	0.20%	0.18%	0.17%
System Volatility $\eta\sigma_\eta$	4.20%	3.84%	3.71%
Welfare Gains (CE)	–	-0.98%	-1.92%
Change in Bank Valuations (pct)	–	-5.95%	-8.19%

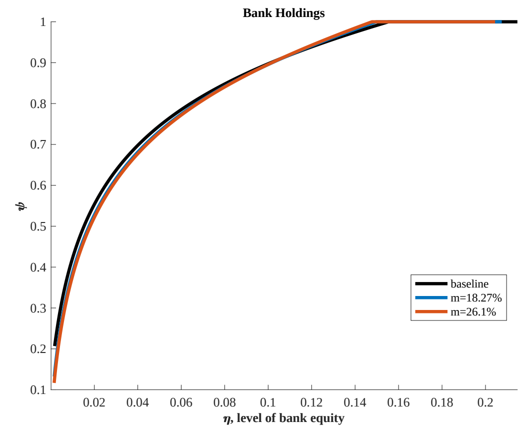
B.2 Additional Figures

B.2.1 Baseline Calibration

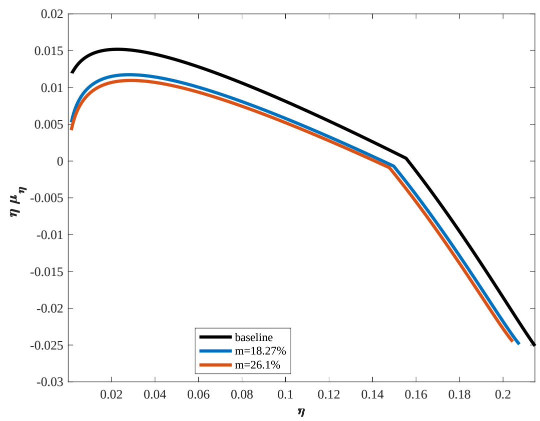
Figure 5 plots the equilibrium variables when increasing digital currency to $m = 18.27\%$ and $m = 26.1\%$.



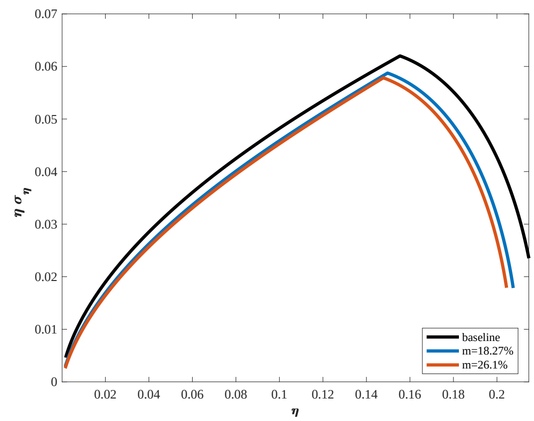
(a) Asset Price $Q(\eta)$



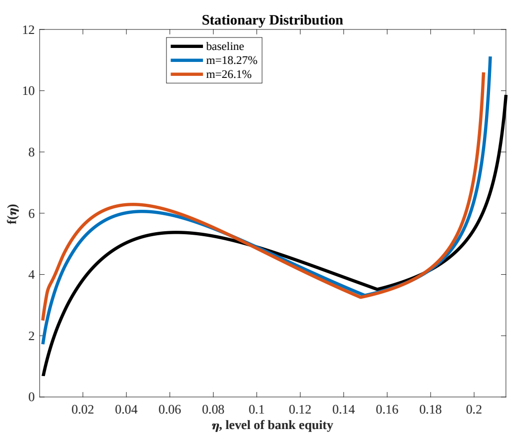
(b) Bank holdings $\psi(\eta)$



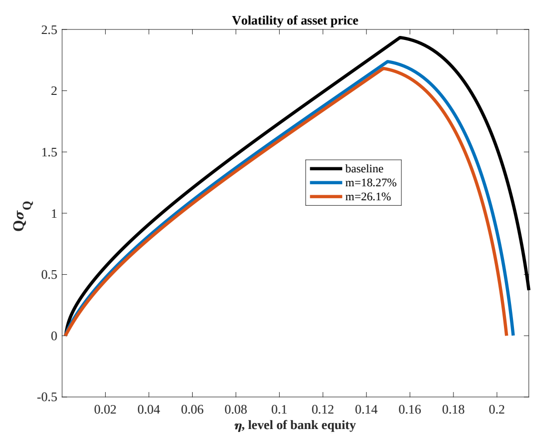
(c) Systemic drift $\eta\mu_\eta$



(d) Systemic volatility $\eta\sigma_\eta$



(e) Stationary Density $f(\eta)$



(f) Endogenous price volatility $Q\sigma_Q$

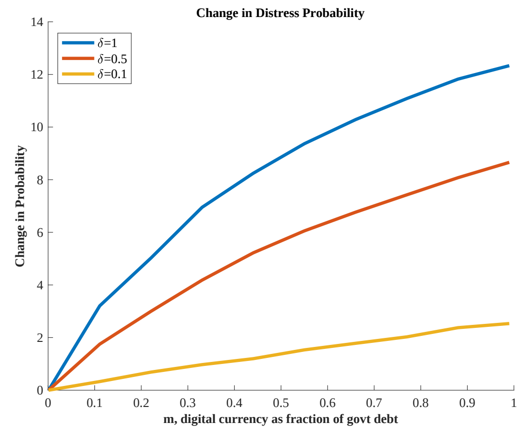
Figure 5: Consequences of digital currency for equilibrium prices, allocations, evolutions, and stationary density in baseline economy. In each panel, the horizontal axis is the state variable η, which is the ratio of banks' book equity to the total value of capital.

Dependence on m and monotonicity of positive results We now present figures that show the equilibrium consequences of converting a fraction $m \in (0,1)$ of government debt to digital currency over the entire range. The main takeaway is that our positive results are generally monotonic in m: if digital currency increases (decreases) a variable X, then the X increases (decreases) by even more for higher m. Some of the relationships are close to linear. Welfare, in contrast, is concave in m, and hump-shaped for intermediate values of κ.

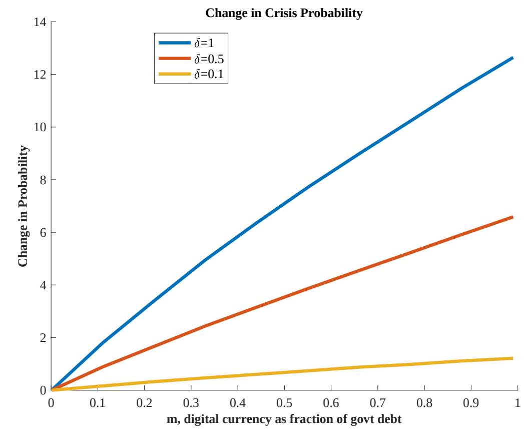
Importantly, for lower values of m issuance of digital currency is profitable (when the spread DCS_t exceeds the cost κ) while for some higher values of m issuance of digital currency loses money. We use the stationary distribution $f(\eta)$ to calculate averages of variables and the probability of crises and distress.

When we adjust δ, we also adjust the digital currency intermediation cost to 100δ bps so that δ adjusts for how digital currency spreads relate to deposit spreads. In this way, the cost κ is still near the average digital currency spread for m in the neighborhood of our benchmark economy.

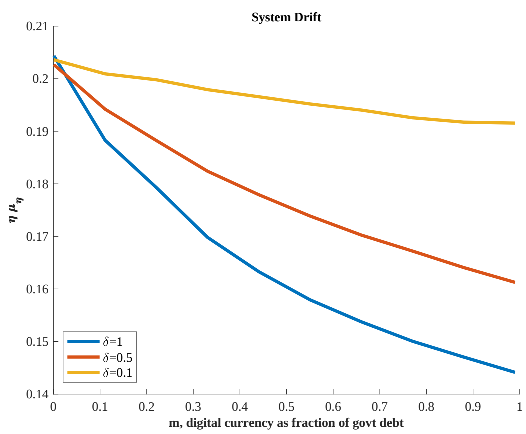
Baseline elasticity with robustness to δ Figure 6 presents our benchmark quantitative results with $\varepsilon = 5.3$ (the elasticity of substitution between bank deposits and cash), varying the preference parameter δ. We set the issuance cost $\kappa = \delta/100$ so that the average spread on digital currency is close to the cost (recall that the average level of deposit spreads without digital currency is 108bps). For low levels of m the spread on digital currency is high (digital currency is scarce) and so issuance is profitable.



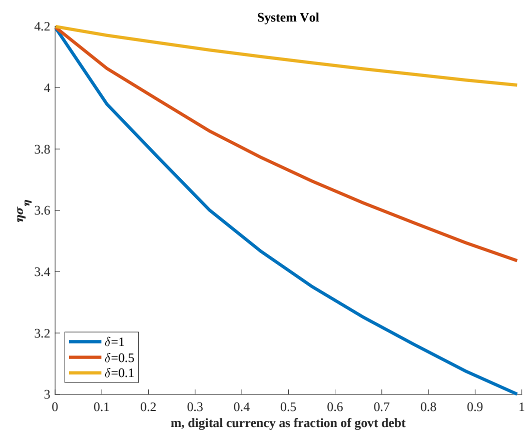
(a) Distress ($\psi < 1$) Probability



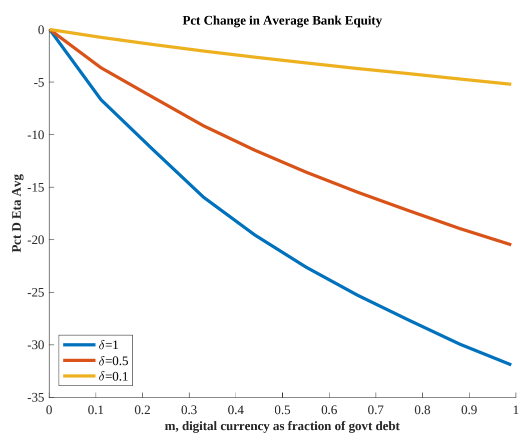
(b) Crisis ($\psi < 0.5$) Probability



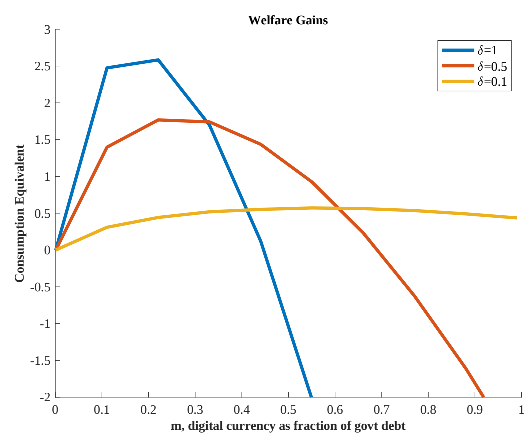
(c) Systemic drift $\eta\mu_\eta$



(d) Systemic volatility $\eta\sigma_\eta$



(e) Average Bank Equity η

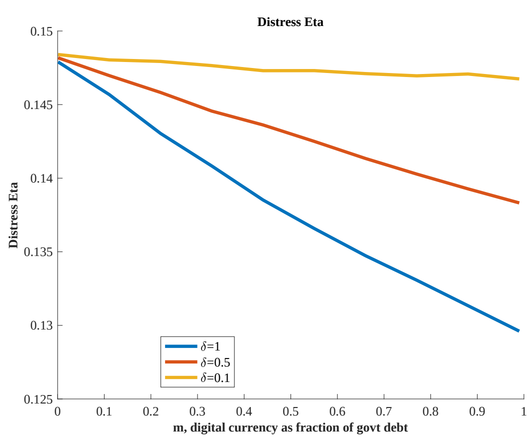


(f) Welfare

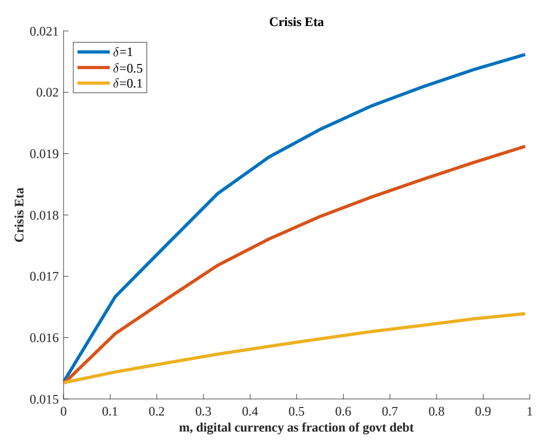
Figure 6: The effects of digital currency on financial stability and welfare with $\varepsilon = 5.3$, varying the digital currency preference δ. Intermediation cost set to $\kappa = \delta/100$.

Several robust results emerge. First, digital currencies harm the various "bottom line metrics" for financial stability. Digital currency increases the probability of crises (panel (b)) and the probability of distress (panel (a)). The dynamics of bank equity feature lower average volatility (d) but lower average drift (c), reflecting a greater challenge rebuilding equity after losses because deposit spreads are lower. As a result, the average level of bank equity decreases (panel (e)). This also reflects a decrease in the upper boundary $\overline{\eta}$ that occurs with digital currency. Thus, while the volatility of the financial system decreases, the overall stability suffers, reflected in higher probabilities of crises and distress and lower levels of equity. Not shown is that asset price volatility decreases monotonically with digital currency issuance and asset prices increase.

Figure 7 provides additional insight into the effects of digital currency on financial stability. The panels plot the thresholds for η determining cutoffs for distress and crises. The distress (crisis) threshold is the value of η such that for η below the threshold bank holdings satisfy $\psi < 1$ ($\psi < 0.5$). Digital currency increases the crisis threshold, implying that bank holdings fall below $\psi = 0.5$ earlier in the cycle than would otherwise occur. Surprisingly, digital currency increases the distress threshold, implying that banks hold the full capital stock for longer. Nonetheless, the *probability* of distress actually increases even though banks hold $\psi = 1$ in some cases when they otherwise would not have.



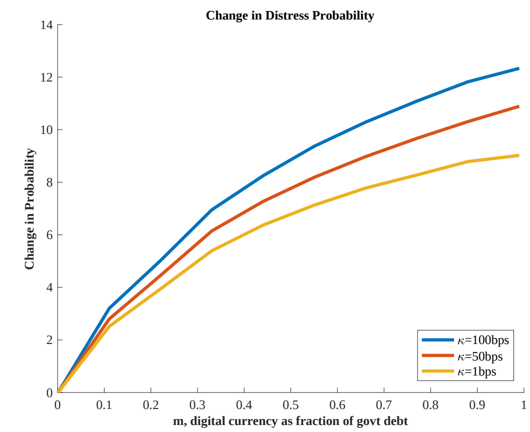
(a) Distress ($\psi < 1$) Threshold



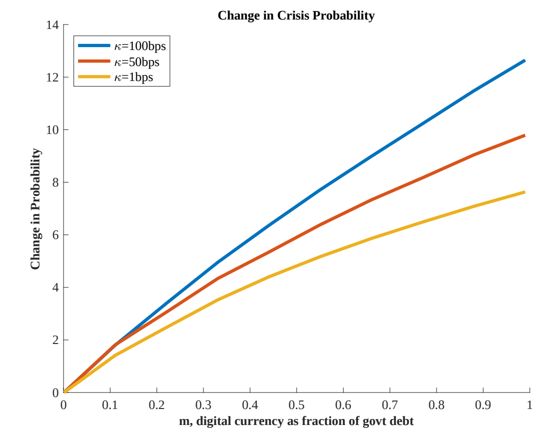
(b) Crisis ($\psi < 0.5$) Threshold

Figure 7: Changes in fire sale thresholds with $\varepsilon = 5.3$, varying the digital currency preference δ. Intermediation cost set to $\kappa = \delta/100$. The distress (crisis) threshold is the value of η such that for η below the threshold bank holdings satisfy $\psi < 1$ ($\psi < 0.5$).

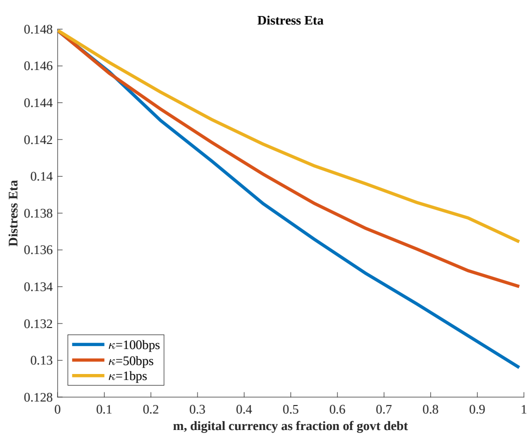
Intermediation costs Our positive results are quantitatively, but not qualitatively, sensitive to the cost κ of issuing digital currency. Decreasing the cost κ slightly mitigates the comparative dynamics (e.g., probability of crisis increases but by slightly less). In contrast, the welfare results depend critically on κ. Figures 8-9 plot results with $\varepsilon = 5.3$, $\delta = 1$ and varying the cost κ. Unsurprisingly, the welfare gains from digital currency are higher with lower costs.



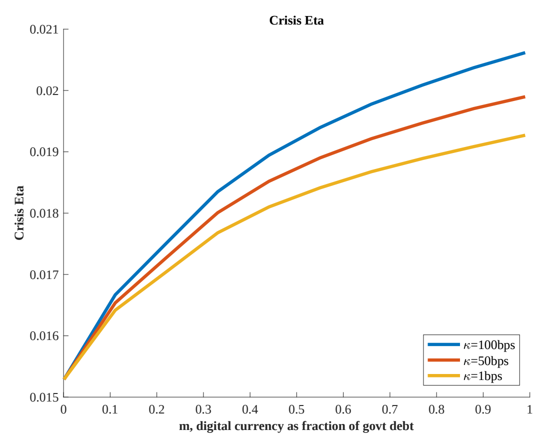
(a) Distress ($\psi < 1$) Probability



(b) Crisis ($\psi < 0.5$) Probability

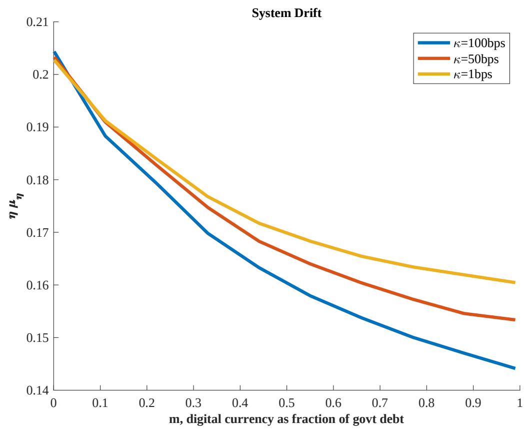


(c) Distress ($\psi < 1$) Thresold

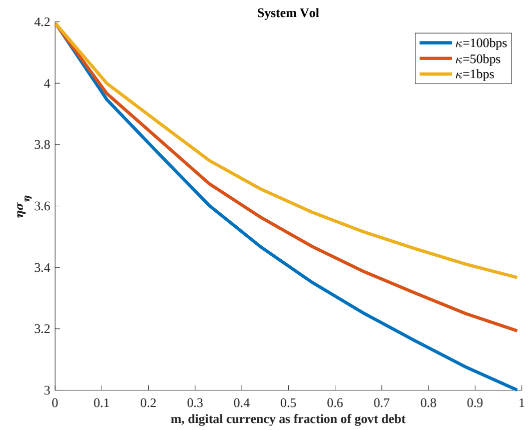


(d) Crisis ($\psi < 0.5$) Threshold

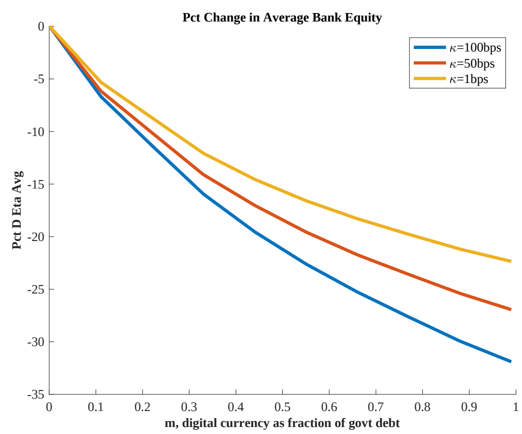
Figure 8: Changes in financial stability and fire sale thresholds with $\varepsilon = 5.3$, varying the cost κ.



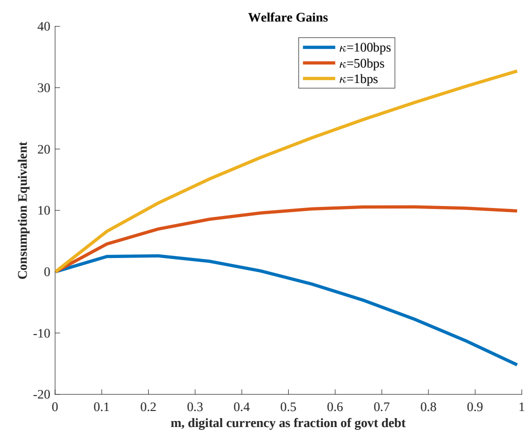
(a) Systemic drift $\eta\mu_\eta$



(b) Systemic volatility $\eta\sigma_\eta$



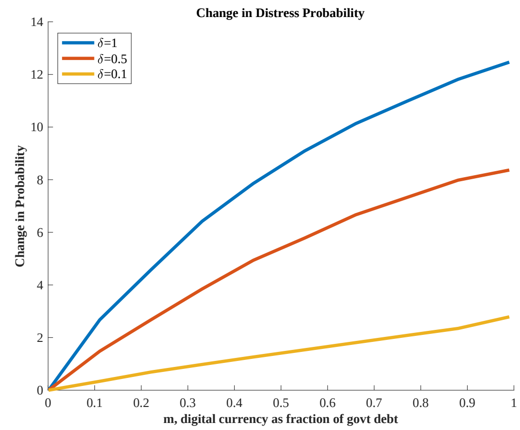
(c) Average Bank Equity η



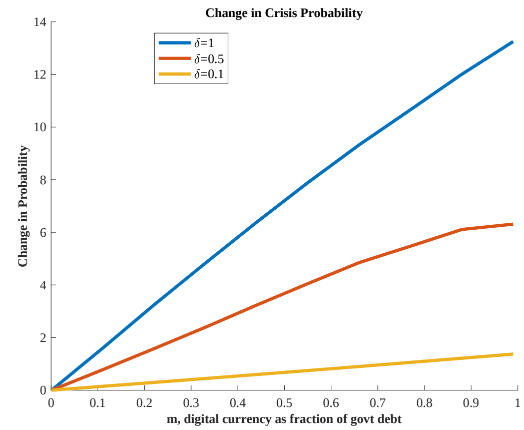
(d) Welfare

Figure 9: Changes in financial stability and welfare with $\varepsilon = 5.3$, varying the cost κ.

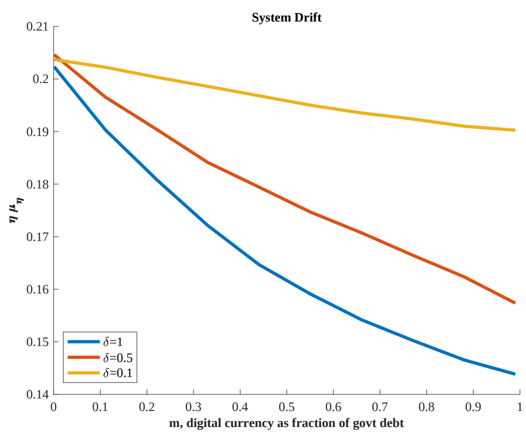
High Elasticity Figures 10-11 present our results with the higher elasticity of $\varepsilon = 53$, varying the preference parameter δ. Again, positive results are monotonic in m. Welfare gains are much smaller and welfare is generally decreasing with a high elasticity between deposits and digital currency.



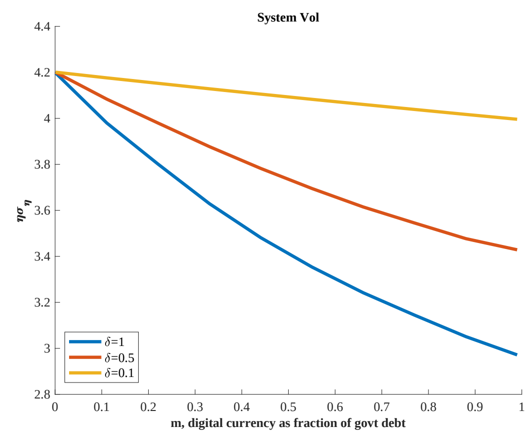
(a) Distress ($\psi < 1$) Probability



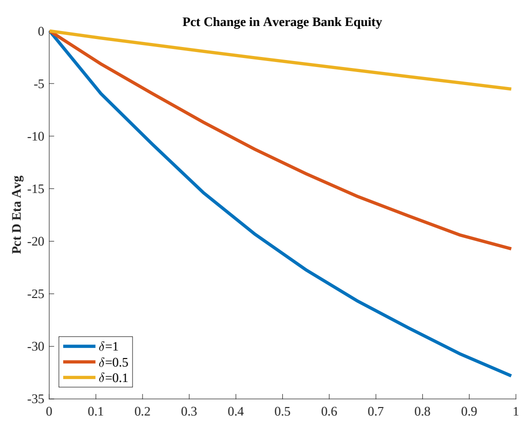
(b) Crisis ($\psi < 0.5$) Probability



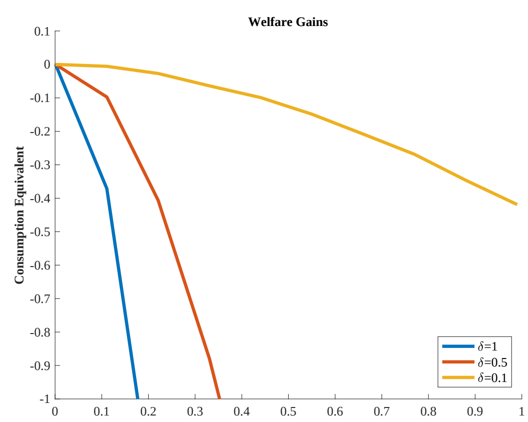
(c) Systemic drift $\eta \mu_\eta$



(d) Systemic volatility $\eta \sigma_\eta$

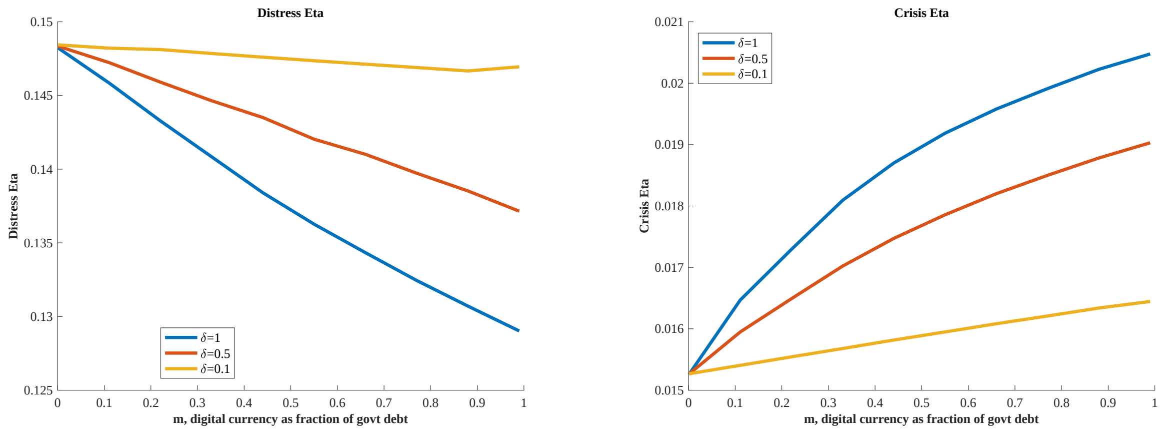


(e) Average Bank Equity η



(f) Welfare

Figure 10: The effects of digital currency on financial stability and welfare with $\varepsilon = 53$, varying the digital currency preference δ. Intermediation cost set to $\kappa = \delta/100$.



(a) Distress ($\psi < 1$) Threshold

(b) Crisis ($\psi < 0.5$) Threshold

Figure 11: Changes in fire sale thresholds with $\varepsilon = 53$, varying the digital currency preference δ. Intermediation cost set to $\kappa = \delta/100$. The distress (crisis) threshold is the value of η such that for η below the threshold bank holdings satisfy $\psi < 1$ ($\psi < 0.5$).

C Alternative Calibration

This section presents our alternative calibration strategy that considers the financial sector more akin to shadow banking activities. The main results of our paper are qualitatively robust in this alternative—though very different—calibration.

In our benchmark calibration we considered the financial sector as primarily corresponding to the banking sector: deposit spreads are high, and output losses from disintermediation are significant. In this section we suppose that financial intermediaries are shadow banks (or even money market funds), holding financial securities and issuing short-term liabilities that have money-like qualities. This calibration deviates from the benchmark one in two key ways.

First, we consider significantly lower deposit spreads, treating deposits as more akin to shadow banking activity. van Binsbergen, Diamond, and Grotteria (2022) find convenience yields on Treasuries in the 35-45bps range, which we use as a target for the convenience yields for financial sector liabilities.

Second, we assume that banks' only advantage comes from their ability to issue liquid liabilities. There are two ways to think about this assumption. First, this is consistent with the evidence in Egan, Lewellen, and Sunderam (2021) who find that 70% of the variation in bank value comes from deposit creation, not asset issuance, suggesting that bank value is driven by their liabilities and not by investment advantages. Second, this would be consistent with shadow banking activity in which institutions hold tradable securities to issue short-term debt—presumably without enhancing the payoffs of the financial securities they hold. Accordingly, we also set the volatility of returns lower, though still within the values found in the literature.

C.1 Calibration and Model Dynamics

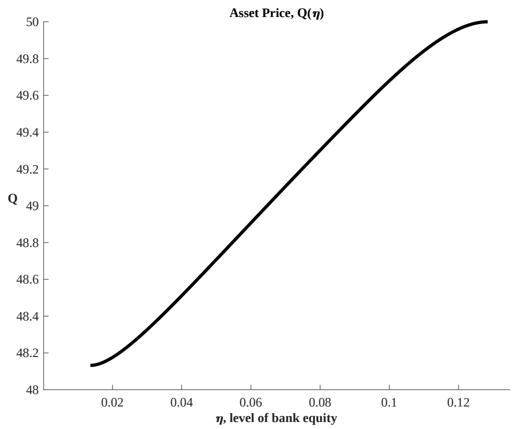
Parameters We target an adjusted value for average deposit spreads, but we continue to target average leverage of 10 and crisis probability of 3. Because banks no longer have a productivity advantage, the Sharpe ratio in the model is substantially lower. It is no longer a target, but we provide the value below for comparison (the resultant Sharpe is comparable to what Drechsler et al., 2018, produce in their calibration). Table 8 provides the parameters used in this calibration as well as the empirical moments.

Model Dynamics Figures 12-13 plot the behavior of the model with this alternative calibration. There are a few minor differences compared to the benchmark calibration but otherwise the model dynamics are quite comparable. The range of η is shorter because the issuance costs are significantly lower, which provides incentive to issue equity at a higher η and to pay out dividends at a lower η.

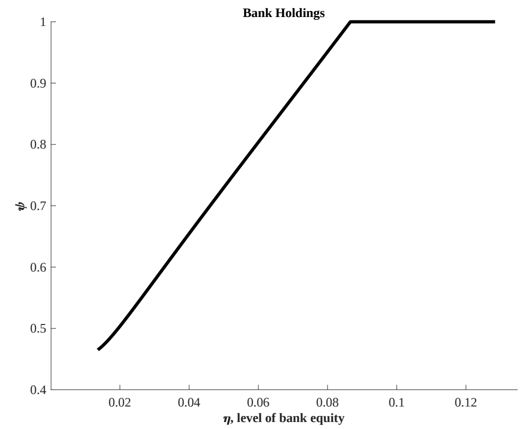
C.2 Quantitative Results with Digital Currency

We now consider the effects of digital currency in this alternative calibration. Because deposit spreads are substantially lower, the spreads on digital currency are substantially lower and we therefore impose a lower cost of issuing digital currency, setting $\kappa = 30$bps instead of 100bps as before. Table 9 considers the same exercise of setting $m = 18.27\%$ and $m = 26.1\%$ within this "shadow banking economy."

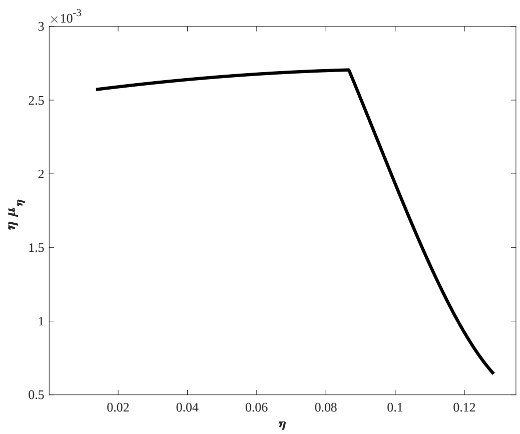
The positive results in this economy are even more quantitatively significant. The probabilities



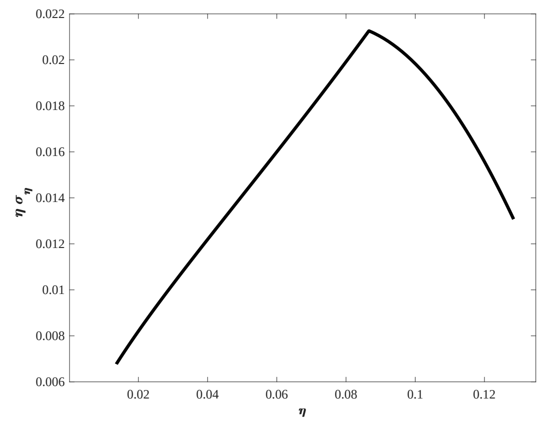
(a) Asset Price $Q(\eta)$



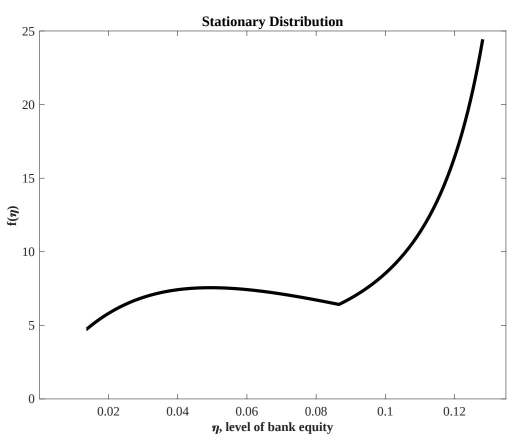
(b) Bank holdings $\psi(\eta)$



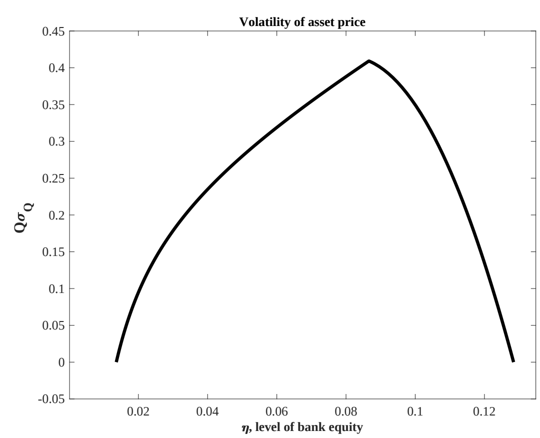
(c) Systemic drift $\eta\mu_\eta$



(d) Systemic volatility $\eta\sigma_\eta$

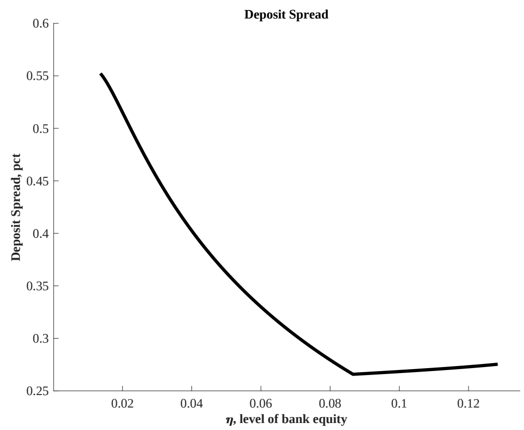


(e) Stationary Density $f(\eta)$

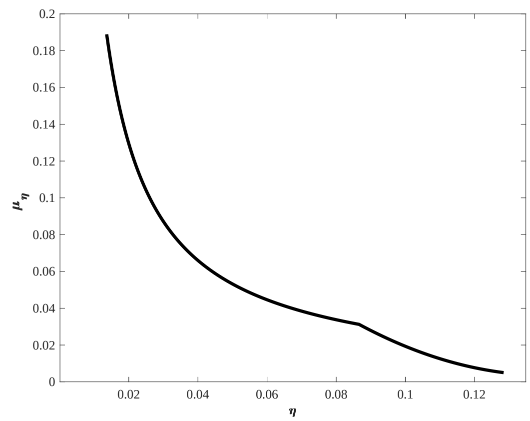


(f) Endogenous price volatility $Q\sigma_Q$

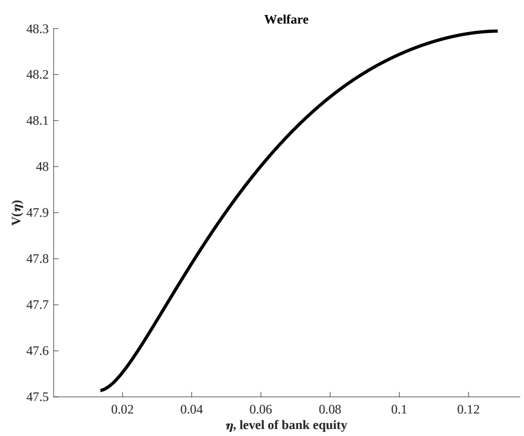
Figure 12: Alternative “shadow banking” calibration: equilibrium prices, allocations, evolutions, and stationary density. In each panel, the horizontal axis is the state variable η, which is the ratio of banks’ book equity to the total value of capital.



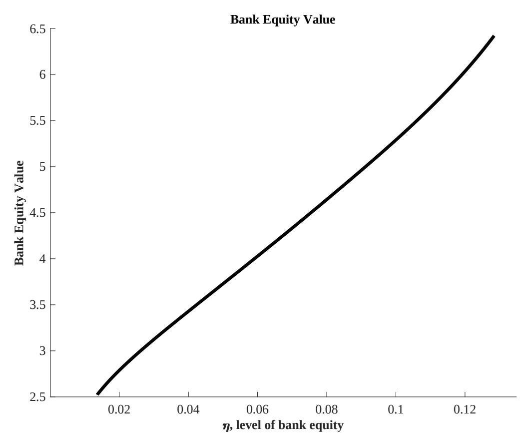
(a) Deposit Spreads



(b) System Drift μ_η



(c) Household Welfare



(d) Bank Valuations

Figure 13: Alternative “shadow banking” calibration: deposit spreads, system drift, welfare, and bank valuations.

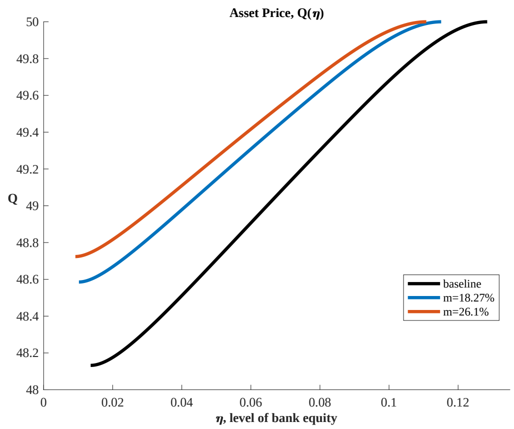
Table 8: Model parameters in alternative calibration.

Description	**Parameter**	**Value**
	Parameters Values	
Discount Rate	r	2%
Growth Rate	$g_b = g_h$	2%
Fundamental Volatility	σ	1.5%
Liquidity Preference	β	0.12
Equity Issuance Cost	γ	2.85
Bank Transfers	$\mathscr{T}_b$	1.5%
Government Debt	q^D	50

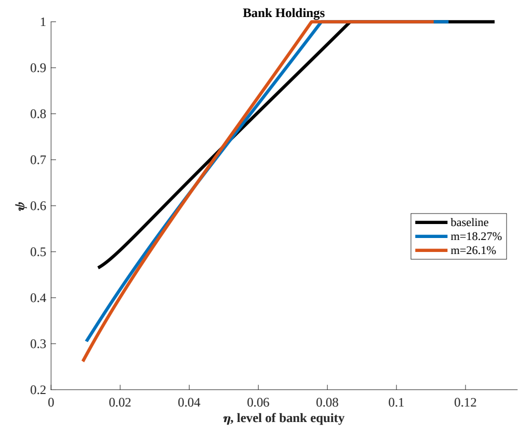
	Empirical Moments	
Empirical Moment	**Data**	**Model**
Average Deposit Rate	35 bps	32.3 bps
Average Bank Leverage	10	11.61
Prob. of Crisis ($\psi < 50\%$)	3%	3.05%
Average Sharpe Ratio (not a target)	48%	13.28%

of crises and distress increase dramatically and bank valuations fall by 3 times as much as they did in the previous calibration. Qualitatively the results are all similar. In terms of welfare, this economy continues to feature welfare gains, though the quantitative significance are much smaller.

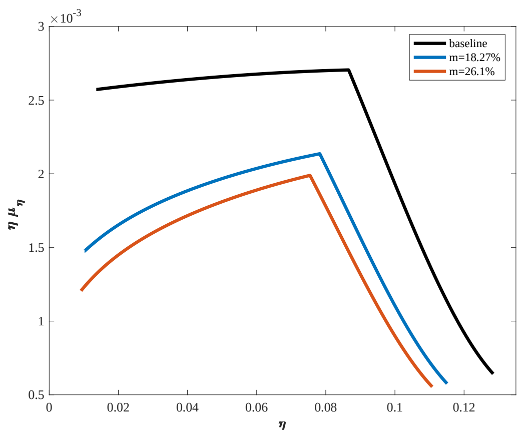
Figure 14 plots the changes in equilibrium when introducing digital currency using the alternative calibration.



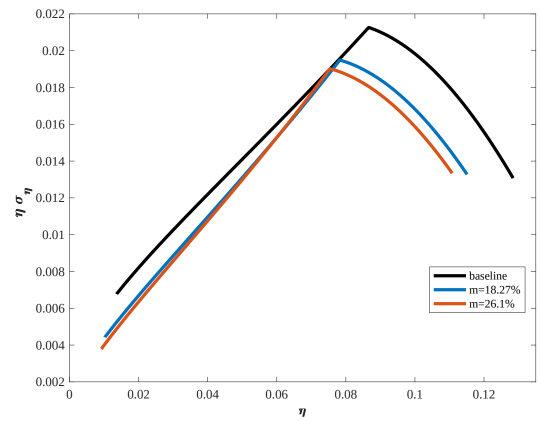
(a) Asset Price $Q(\eta)$



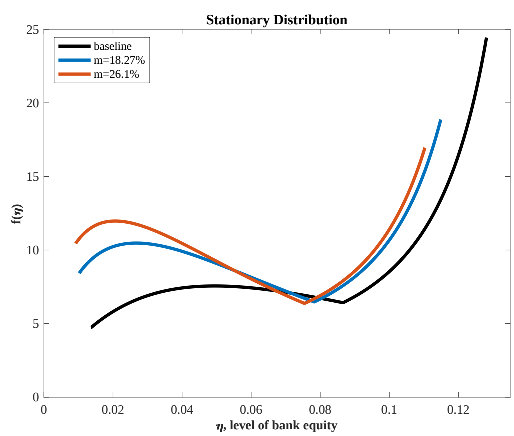
(b) Bank holdings $\psi(\eta)$



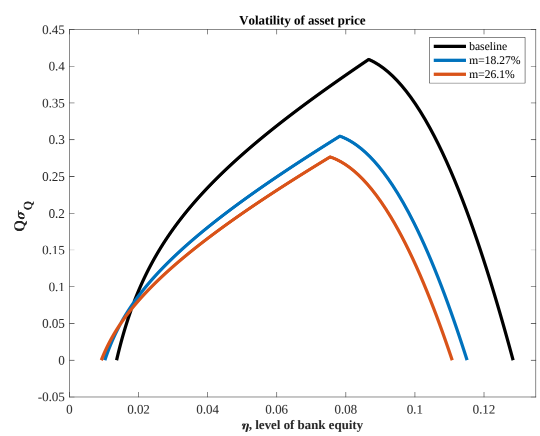
(c) Systemic drift $\eta\mu_\eta$



(d) Systemic volatility $\eta\sigma_\eta$



(e) Stationary Density $f(\eta)$

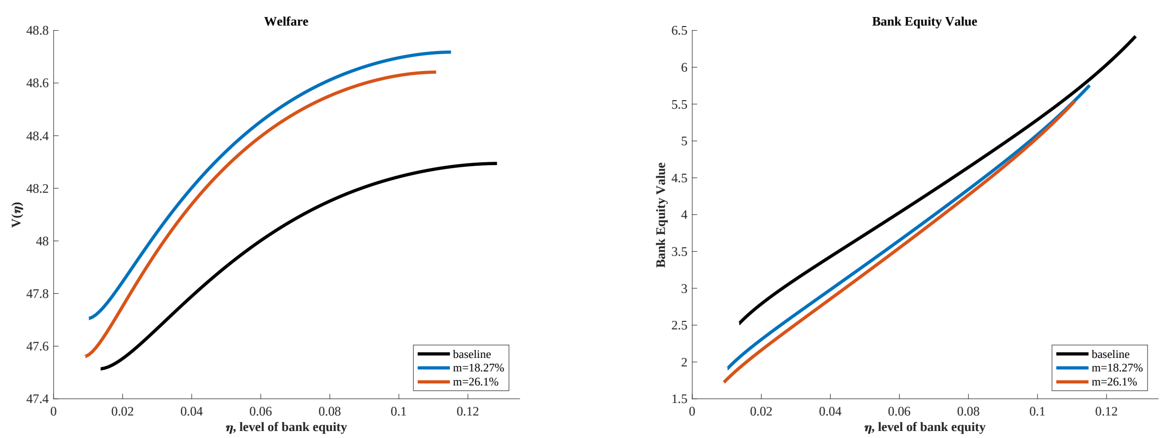


(f) Endogenous price volatility $Q\sigma_Q$

Figure 14: Alternative "shadow banking" calibration: effects of digital currency on equilibrium prices, allocations, evolutions, and stationary density in baseline economy. In each panel, the horizontal axis is the state variable η, which is the ratio of banks' book equity to the total value of capital.

Table 9: Model outcomes with alternative calibration, $\varepsilon = 5.3$. Averages of variables are computed by integrating the moment of interest with respect to the stationary density.

Moment	**Baseline** $m = 0$	$m = 18.27\%$	$m = 26.1\%$
Average Deposit Rate	32.3 bps	26.4 bps	25.1 bps
Average Bank Leverage	11.61	13.24	13.83
Prob. of Crisis ($\psi < 50\%$)	3.05%	16.99%	22.31%
Prob. of Distress ($\psi < 100\%$)	49.60%	60.27%	64.12%
Average Sharpe Ratio	13.28%	13.29%	13.36%
Output Loss in Crisis	-0.00%	0.00%	-0.00%
Average Bank Holdings ψ	86.64%	78.00%	74.52%
Average Bank Equity η	8.11%	6.44%	5.88%
Asset Price Volatility σ_Q	0.49%	0.35%	0.31%
System Drift $\eta\mu_\eta$	0.20%	0.16%	0.15%
System Volatility $\eta\sigma_\eta$	1.57%	1.34%	1.26%
Average Asset Price Q	49.28	49.35	49.38
Welfare Gains (CE)	–	0.61%	0.37%
Change in Bank Valuations (pct)	–	-18.95%	-25.29%



(a) Welfare (b) Bank Valuation

Figure 15: Alternative "shadow banking" calibration: effects of digital currency on household welfare and bank valuations.

C.3 Welfare and Optimal Level of Digital Currency

Table 10 provides the optimal and equilibrium supply of digital currency with and without financial frictions, varying the elasticity of substitution ε and the preference for digital currency δ. In this calibration, the optimal level of digital currency m_1^* in the model with frictions is always less than the optimal level in the first-best m_{FB}^* as well as what would be privately produced in competitive equilibrium $\hat{m}_1$. In the shadow banking calibration, the negative externalities from financial stability always outweigh the built-in positive aggregate externality from digital currency issuance.

Table 10: Optimal and equilibrium supply of digital currency with and without financial frictions in the "shadow banking" calibration. This table provides the competitive levels $\hat{m}$ together with the optimal levels m^*. The subscript FB denotes the first-best and the subscript 1 denotes the model with frictions.

	First-Best		*Frictions*	
Currency preference	$\hat{m}_{FB}$	m_{FB}^*	$\hat{m}_1$	m_1^*
Baseline elasticity $\varepsilon = 5.3$				
$\delta = 1.25$	11.2%	14.9%	19.6%	11.1%
$\delta = 1.00$	12.5%	17.0%	22.1%	12.8%
$\delta = 0.50$	16.9%	24.8%	29.1%	18.3%
Higher elasticity $\varepsilon = 53$				
$\delta = 1.00$	$\approx 0\%$	0.47%	7.8%	0.47%
$\delta = 0.50$	$\approx 0\%$	0.53%	8.2%	0.52%

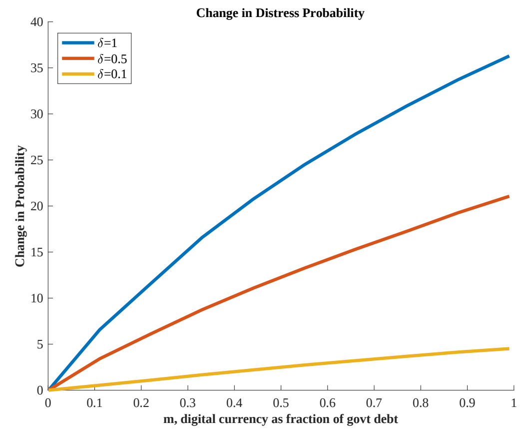
The shadow banking calibration suggests that the potential welfare gains from digital currency are small, the optimal level of digital currency would be quite low, and the consequences for financial stability are severe.

The careful reader may wonder: if the shadow banking calibration gives traditional intermediaries the smallest advantages (lower deposit spreads and no productivity gains), then why are the welfare gains from CBDC so low (and the likelihood for welfare losses high)? Our modeling assumption still privileges traditional liquidity provision. In our model, financial intermediaries produce liquidity by holding real risky assets. The "cost" of intermediation arises due to financial frictions (the need to avoid bankruptcy) but there are otherwise no costs of intermediation (banks pay a cost $\mathscr{T}_b$, but this is not a resource cost that decreases output as does ι). In contrast, digital currencies are issued by holding risk-free assets at an assumed cost. Even though the costs κ can be quite small, they are still non-negligible.

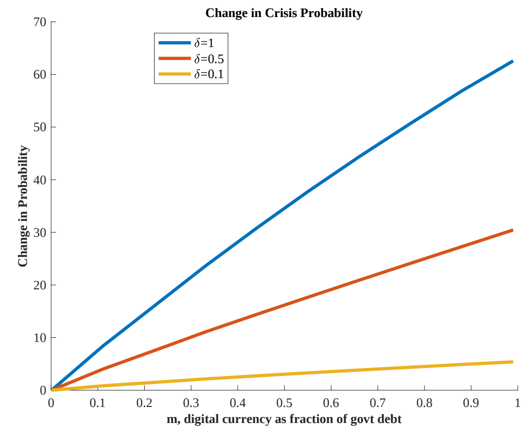
How should we interpret the costs κ? In the case of a CBDC, this is likely capturing risks to government funding that occur when the government substantially shortens the maturity of its borrowing, and could potentially reflect inflationary risks. In the case of privately issued digital currency, the cost could reflect the real costs of maintaining the infrastructure of a stable coin (managing off-chain collateral) and potential protocol risks. It's possible that the costs of issuing digital currency are actually much smaller than we've assumed. In that case, our positive results maintain, but the welfare gains of issuing digital currency will natural increase.

C.4 Robustness

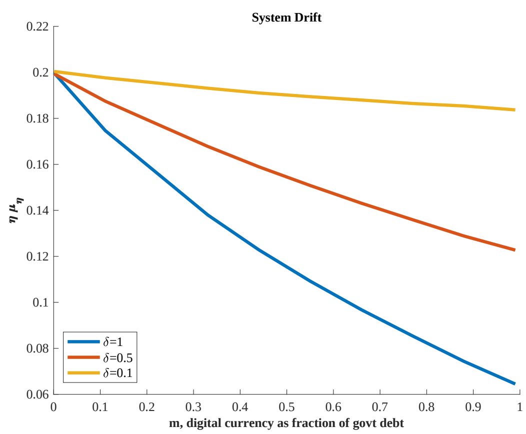
Range of m with baseline elasticity and robustness to δ Figure 16 considers our alternative calibration and plots outcomes varying $m \in (0,1)$, with $\varepsilon = 5.3$. The qualitative results match the baseline calibration, but the quantitative consequences can be quite different. In particular, the probabilities of crises and distress increase by much more in this calibration. Welfare gains are significantly smaller as well.



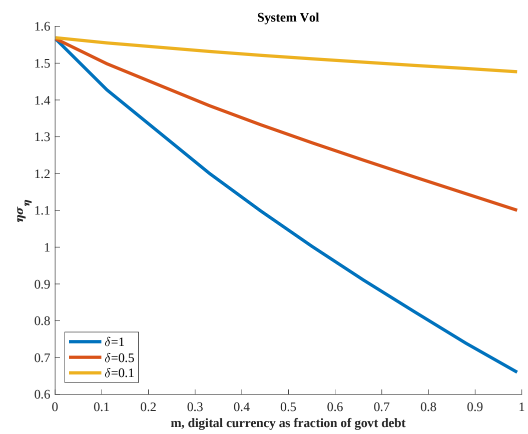
(a) Distress ($\psi < 1$) Probability



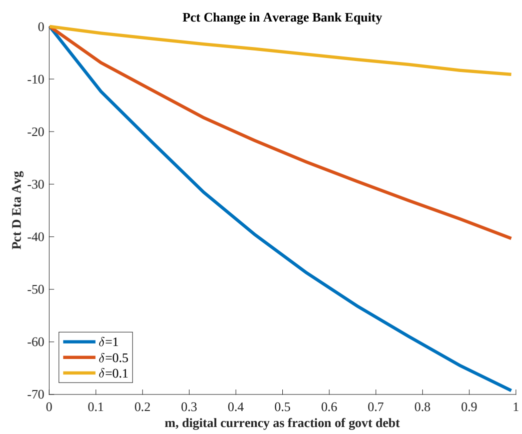
(b) Crisis ($\psi < 0.5$) Probability



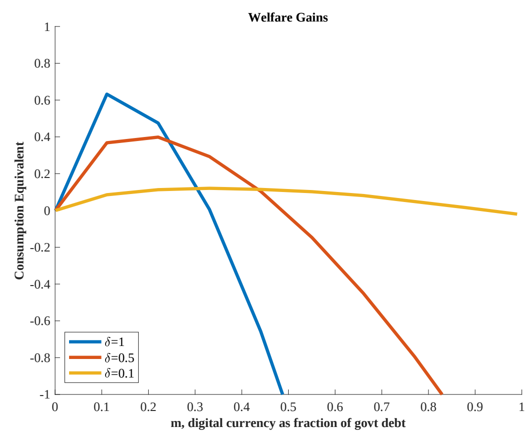
(c) Systemic drift $\eta\mu_\eta$



(d) Systemic volatility $\eta\sigma_\eta$

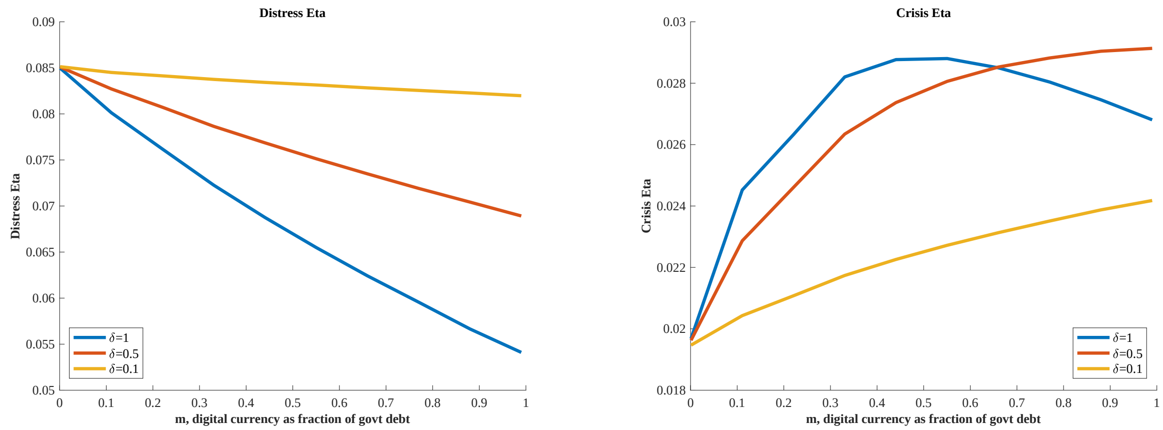


(e) Average Bank Equity η



(f) Welfare

Figure 16: Alternative "shadow banking" calibration: the effects of digital currency on financial stability and welfare with $\varepsilon = 5.3$, varying the digital currency preference δ. Intermediation cost set to $\kappa = 0.3 * \delta/100$.

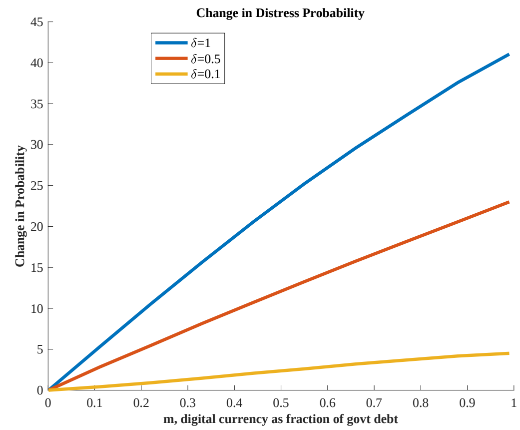


(a) Distress ($\psi < 1$) Threshold

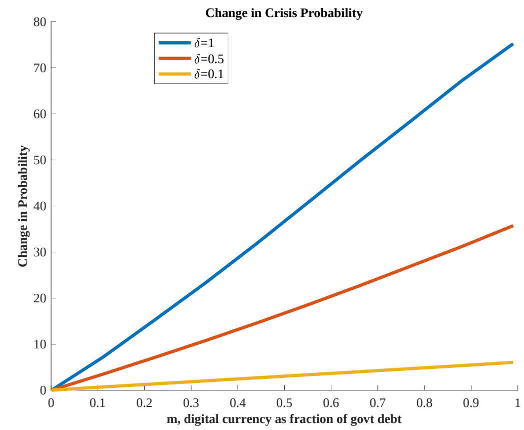
(b) Crisis ($\psi < 0.5$) Threshold

Figure 17: Alternative "shadow banking" calibration: changes in fire sale thresholds with $\varepsilon = 5.3$, varying the digital currency preference δ. Intermediation cost set to $\kappa = 0.3 * \delta/100$. The distress (crisis) threshold is the value of η such that for η below the threshold bank holdings satisfy $\psi < 1$ ($\psi < 0.5$).

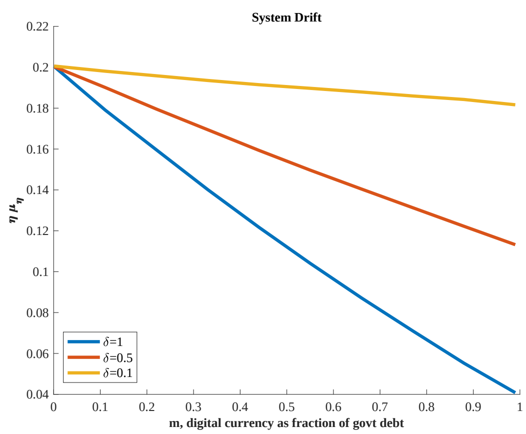
Alternative Calibration: Range of *m* with high elasticity Figures 18-19 present our results with the higher elasticity of $\varepsilon = 53$, varying the preference parameter δ in the alternative calibration. Welfare is generally decreasing in m over the entire range.



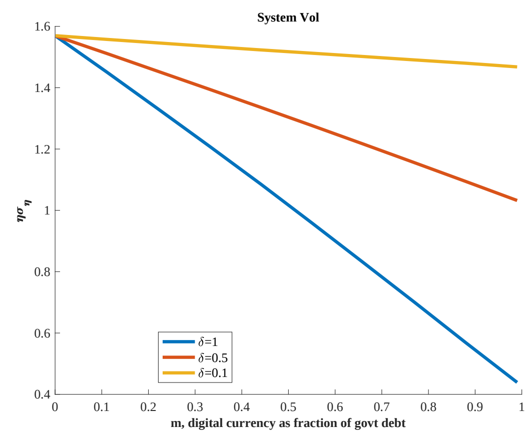
(a) Distress ($\psi < 1$) Probability



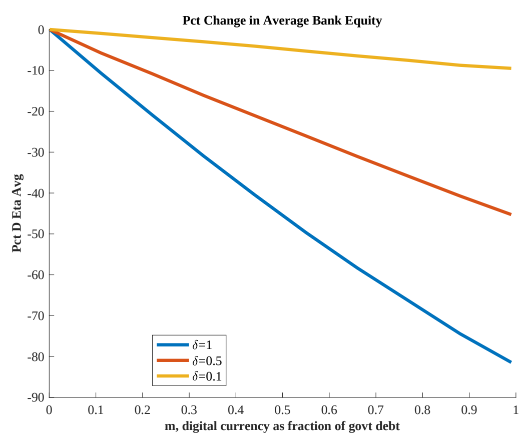
(b) Crisis ($\psi < 0.5$) Probability



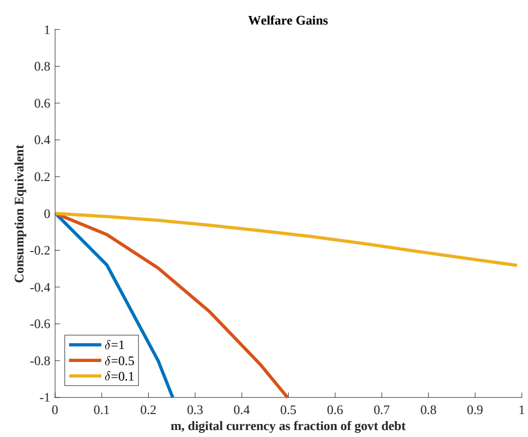
(c) Systemic drift $\eta\mu_\eta$



(d) Systemic volatility $\eta\sigma_\eta$

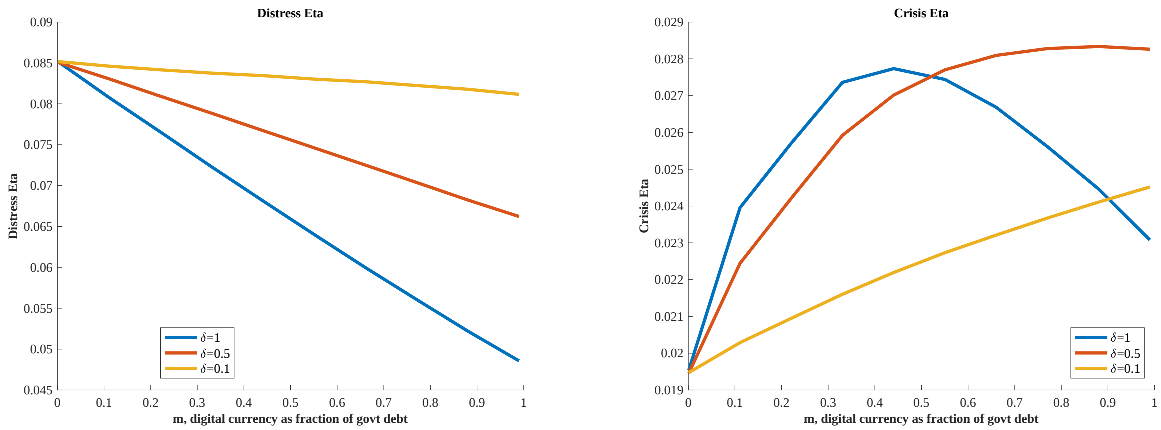


(e) Average Bank Equity η



(f) Welfare

Figure 18: Alternative “shadow banking” calibration: the effects of digital currency on financial stability and welfare with $\varepsilon = 53$, varying the digital currency preference δ. Intermediation cost set to $\kappa = 0.3 * \delta/100$.



(a) Distress ($\psi < 1$) Threshold

(b) Crisis ($\psi < 0.5$) Threshold

Figure 19: Alternative "shadow banking" calibration: changes in fire sale thresholds with $\varepsilon = 53$, varying the digital currency preference δ. Intermediation cost set to $\kappa = 0.3 * \delta/100$. The distress (crisis) threshold is the value of η such that for η below the threshold bank holdings satisfy $\psi < 1$ ($\psi < 0.5$).